Chapter 5 -Year 2 -Pure Maths -Edexcel

Radians

Exam questions with written answers

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www.onlinetutors.xyz

1.

Figure 1

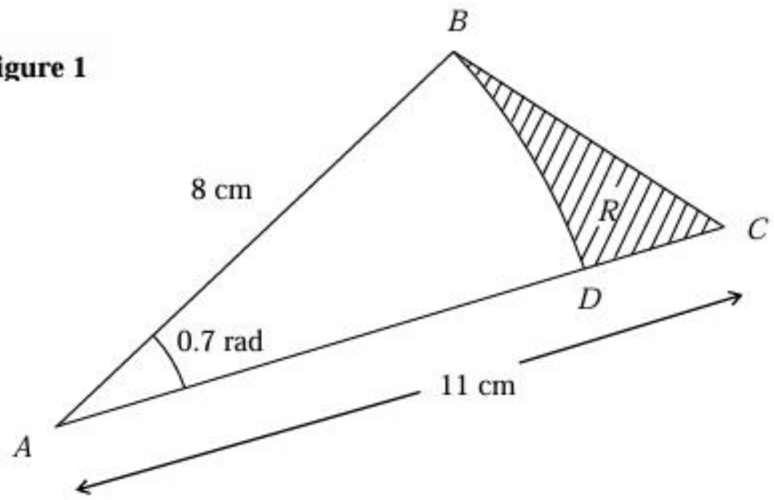


Figure 1 shows the triangle ABC, with $AB = 8$ cm, $AC = 11$ cm and $\angle BAC = 0.7$ radians. The arc BD, where D lies on AC, is an arc of a circle with centre A and radius 8 cm. The region R, shown shaded in Figure 1, is bounded by the straight lines BC and CD and the arc BD.

Find

(*a*) the length of the arc BD, **(2)**

(*b*) the perimeter of R, giving your answer to 3 significant figures, **(4)**

(*c*) the area of R, giving your answer to 3 significant figures. **(5)**

Answer of number 1 has been done by my year 13 student -Sukhjeet Powar 0n 12thSeotemebr 24

1) a) $BD = 0.7 \times 8 = 5.6$ cm

b) $BDC = 5.6 + 3 + \overrightarrow{BC}$

$= 8.6 + \overrightarrow{BC}$

$a^2 = b^2 + c^2 - 2bc\cos(A)$

$a^2 = 8^2 + 11^2 - 2 \times 88 \cos(0.7 \text{ rad})$

$a^2 = 50.387\ldots$

$a = 7.098$

$8.6 + 7.098\ldots = 15.7$ cm

c) $\frac{1}{2}ab \sin C$

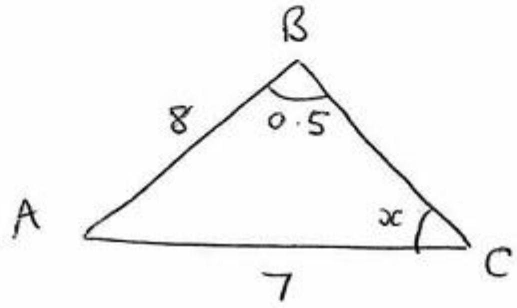
$\hookrightarrow 4 \times 11 \times \sin(0.7) = 28.35\ldots$

2. In the triangle ABC, $AB = 8$ cm, $AC = 7$ cm, $\angle ABC = 0.5$ radians and $\angle ACB = x$ radians.

(*a*) Use the sine rule to find the value of sin x, giving your answer to 3 decimal places. **(3)**

Given that there are two possible values of x,

(*b*) find these values of x, giving your answers to 2 decimal places. **(3)**



a) $$\frac{\sin x}{8} = \frac{\sin 0.5}{7}$$

$$\sin x = \frac{8 \sin 0.5}{7}$$

$$\sin x = 0.548 \quad 3DP$$

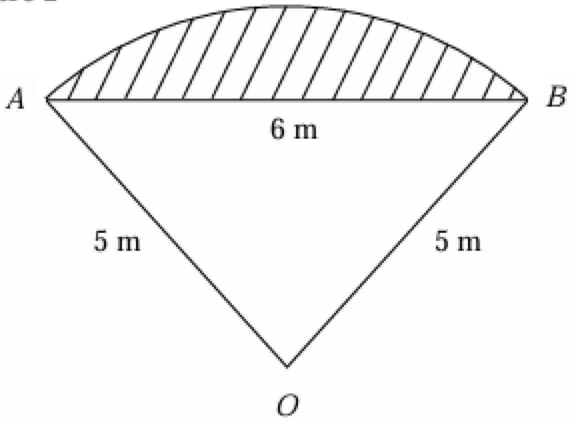
b) $$x = \sin^{-1}(0.548$$

$$= 0.58, \ \pi - 0.58$$

$$= \underline{0.58}, \ \underline{2.56}$$

3.

Figure 2



In Figure 2 OAB is a sector of a circle, radius 5 m. The chord AB is 6 m long.

(*a*) Show that $\cos A\hat{O}B = \frac{7}{25}$. **(2)**

(*b*) Hence find the angle $A\hat{O}B$ in radians, giving your answer to 3 decimal places. **(1)**

(*c*) Calculate the area of the sector OAB. **(2)**

(*d*) Hence calculate the shaded area. **(3)**

a) $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$= \frac{5^2+5^2-6^2}{2(5)(5)}$

$= \frac{14}{50} = \frac{7}{25}$

b) $\cos^{-1}\left(\frac{7}{25}\right) = \underline{\underline{1.287^c}}$

c) $\frac{\theta}{2} \times r^2$

$\frac{1.287}{2} \times 5^2 = 16.09\ m^2$ (2dp)

d) Area of triangle $= \frac{1}{2}(5)(5)\sin(1.287)$

$= 12\ m^2$

Shaded area $= 16.09 - 12 = 4.09\ m^2$ (2dp)

4. **Figure 2**

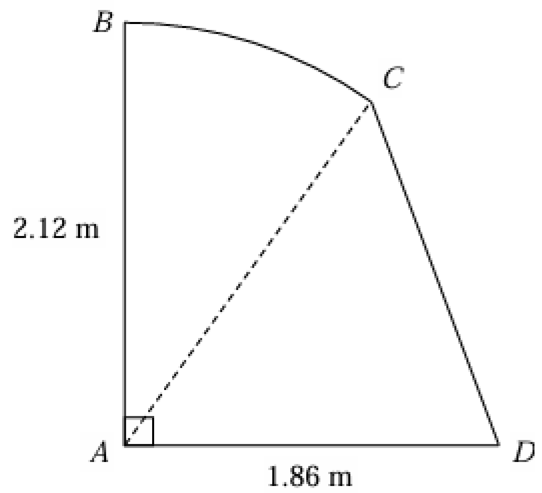


Figure 2 shows the cross-section $ABCD$ of a small shed.

The straight line AB is vertical and has length 2.12 m.

The straight line AD is horizontal and has length 1.86 m.

The curve BC is an arc of a circle with centre A, and CD is a straight line.

Given that the size of $\angle BAC$ is 0.65 radians, find

(*a*) the length of the arc BC, in m, to 2 decimal places, **(2)**

(*b*) the area of the sector BAC, in m^2, to 2 decimal places, **(2)**

(*c*) the size of $\angle CAD$, in radians, to 2 decimal places, **(2)**

(*d*) the area of the cross-section $ABCD$ of the shed, in m^2, to 2 decimal places. **(3)**

Figure 2

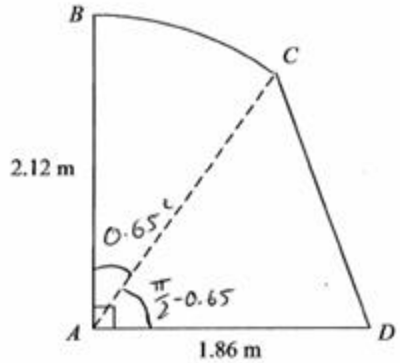


Figure 2 shows the cross-section *ABCD* of a small shed.

The straight line *AB* is vertical and has length 2.12 m.

The straight line *AD* is horizontal and has length 1.86 m.

The curve *BC* is an arc of a circle with centre *A*, and *CD* is a straight line.

Given that the size of $\angle BAC$ is 0.65 radians, find

(*a*) the length of the arc *BC*, in m, to 2 decimal places, **(2)**

(*b*) the area of the sector *BAC*, in m^2, to 2 decimal places, **(2)**

(*c*) the size of $\angle CAD$, in radians, to 2 decimal places, **(2)**

(*d*) the area of the cross-section *ABCD* of the shed, in m^2, to 2 decimal places. **(3)**

a) arclength $= \theta r$
$= 0.65 \times 2.12$
$= \underline{1.378} = \underline{1.38}$ m (2dp)

b) sector area $= \frac{\theta}{2} \times r^2$
$= \frac{0.65}{2} \times 2.12^2$
$= \underline{1.46\text{ m}^2}$ (2dp)

c) $\angle CAD = \pi/2 - 0.65$
$= \underline{0.92^c}$ (2dp)

d) Area of triangle $= \frac{1}{2}(1.86)(2.12)\sin(0.92)$
$= 1.569558817$ m^2

Triangle + Sector $= \underline{3.03\text{ m}^2}$ 2dp

5.

Figure 2

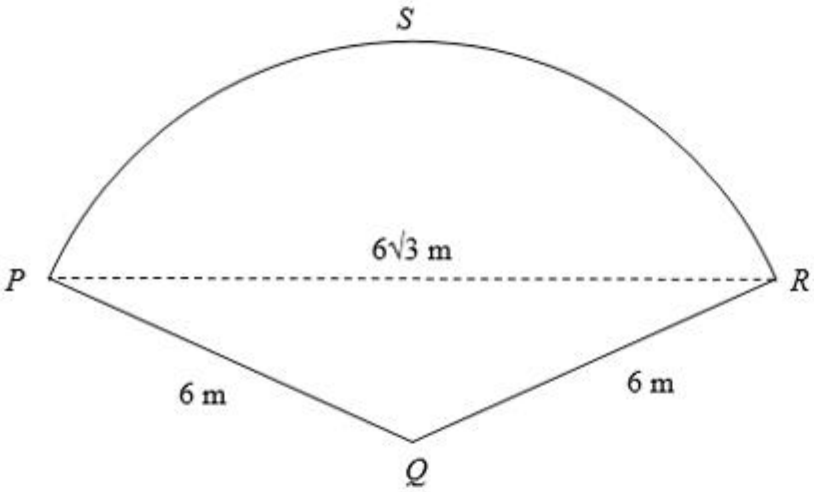


Figure 2 shows a plan of a patio. The patio *PQRS* is in the shape of a sector of a circle with centre *Q* and radius 6 m.

Given that the length of the straight line *PR* is $6\sqrt{3}$ m,

(*a*) find the exact size of angle *PQR* in radians. **(3)**

(*b*) Show that the area of the patio *PQRS* is $12\pi\,\text{m}^2$. **(2)**

(*c*) Find the exact area of the triangle *PQR*. **(2)**

(*d*) Find, in m^2 to 1 decimal place, the area of the segment *PRS*. **(2)**

(*e*) Find, in m to 1 decimal place, the perimeter of the patio *PQRS*. **(2)**

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a) $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

$= \dfrac{6^2+6^2-(6\sqrt{3})^2}{2(6)(6)}$

$\cos A = -\tfrac{1}{2}$

$A = \tfrac{2}{3}\pi$

b) Sector area $= \dfrac{\theta}{2} r^2$

$= \dfrac{\tfrac{2}{3}\pi}{2} \times 6^2$

$= 12\pi \text{ m}^2$

c) $\tfrac{1}{2}(6)(6)\sin\left(\tfrac{2}{3}\pi\right) = 9\sqrt{3} \text{ m}^2$

d) $12\pi - 9\sqrt{3} = \underline{22.1 \text{ m}^2}$ 1dp

e) Arc length $= \theta r$

$= \tfrac{2}{3}\pi \times 6$

$= 4\pi$

Perimeter $= 12 + 4\pi$

$= \underline{24.6 \text{ m}}$ 1dp

6. A circle C has centre $M(6, 4)$ and radius 3.

(a) Write down the equation of the circle in the form

$$(x - a)^2 + (y - b)^2 = r^2.$$ **(2)**

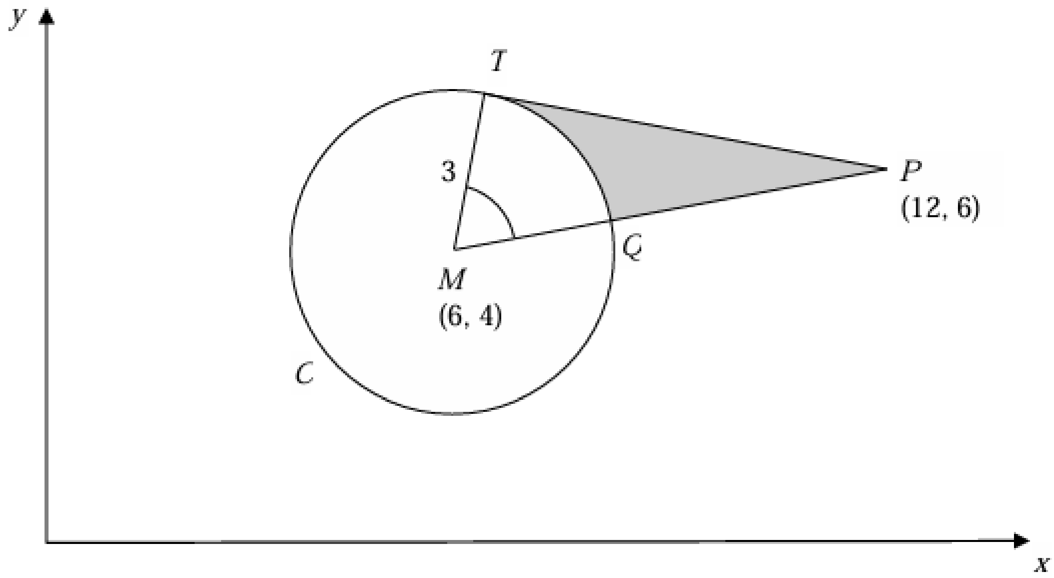


Figure 3 shows the circle C. The point T lies on the circle and the tangent at T passes through the point $P(12, 6)$. The line MP cuts the circle at Q.

(b) Show that the angle TMQ is 1.0766 radians to 4 decimal places. **(4)**

The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ, as shown in Figure 3.

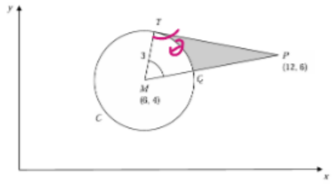
(c) Find the area of the shaded region TPQ. Give your answer to 3 decimal places.

(5)

6. A circle C has centre $M(6, 4)$ and radius 3.

(*a*) Write down the equation of the circle in the form

$$(x-a)^2 + (y-b)^2 = r^2.$$ **(2)**



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The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ, as shown in Figure 3.

(*c*) Find the area of the shaded region TPQ. Give your answer to 3 decimal places. **(5)**

6) a)

$(x-6)^2 + (y-4)^2 = 9$

b) $MP =$

$\sqrt{(12-6)^2 + (6-4)^2}$

$= \sqrt{40} = 2\sqrt{10}$

$\cos\theta = \frac{MT}{MP} = \frac{3}{2\sqrt{10}}$

$\theta = 1.0766$ Radians

c) Area of TMP $= \frac{1}{2} \times 3 \times 2\sqrt{10} \sin(1.0766)$

$= 8.35$

Area of sector $= 0.5 \times 3^2 \times 1.0760 = 4.8446$

Area of TPQ $= 8.35 - 4.8446$

$= 3.507$

7.

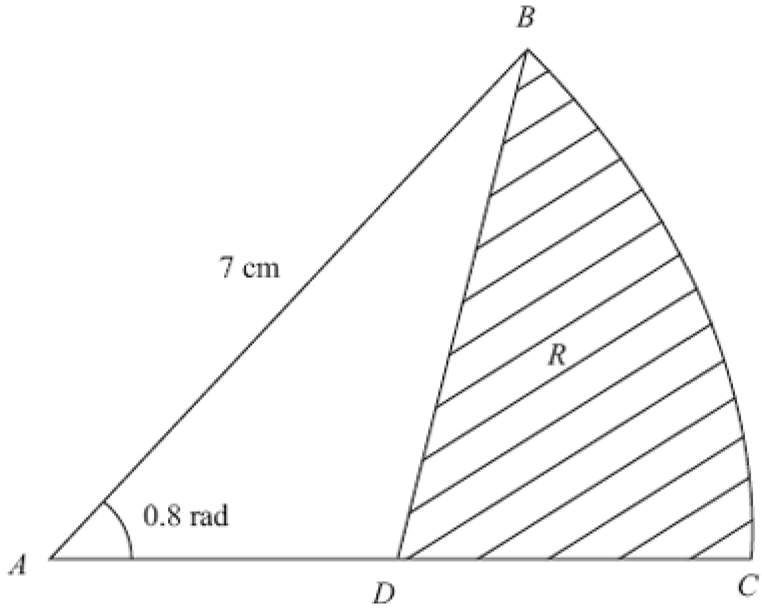


Figure 1

Figure 1 shows ABC, a sector of a circle with centre A and radius 7 cm.

Given that the size of $\angle BAC$ is exactly 0.8 radians, find

(a) the length of the arc BC, **(2)**

(b) the area of the sector ABC. **(2)**

The point D is the mid-point of AC. The region R, shown shaded in Figure 1, is bounded by CD, DB and the arc BC.

Find

(c) the perimeter of R, giving your answer to 3 significant figures, **(4)**

(d) the area of R, giving your answer to 3 significant figures. **(4)**

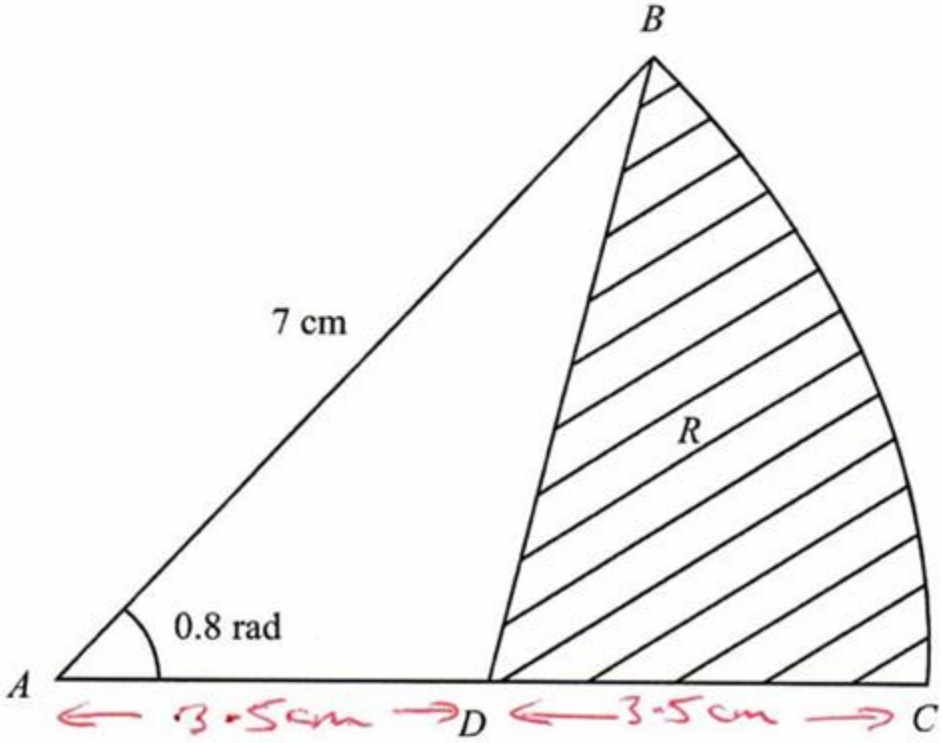
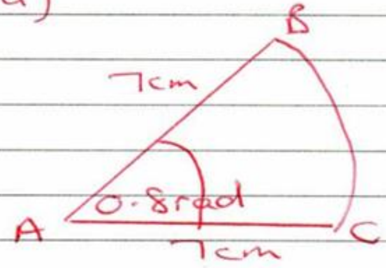


Figure 1

(4)

a)



length of arc BC

$= \frac{0.8}{\cancel{2\pi}} \times \cancel{2\pi} \times 7$

$= 5.6\text{cm}$

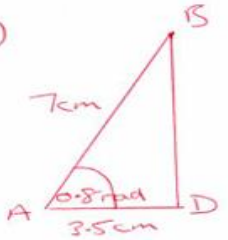
$\boxed{\text{Length of arc} = r\theta}$

7b) Area of sector $= \frac{0.8}{2\pi} \times \pi \times 7^2$

$= 19.6 \text{ cm}^2$

$\boxed{\text{Area of sector} = \frac{1}{2} r^2 \theta}$

$\cos 0.8^c$
→ RADIANS on calculator

c) 

use Cosine rule to find length BD

$BD^2 = 7^2 + 3.5^2 - 2 \times 7 \times 3.5 \times \cos 0.8^c$

$BD^2 = 27.111371\ldots$

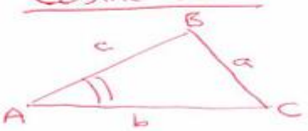
$BD = 5.206858\ldots$

$\therefore$ perimeter of $R = 5.6 + 3.5 + 5.206858$

$= 14.3068\ldots$

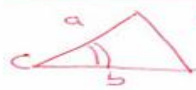
$= 14.3 \text{ cm} \quad (3sf)$

Cosine rule



$a^2 = b^2 + c^2 - 2bc\cos A$

Area Triangle



Area $\Delta = \frac{1}{2}ab\sin C$

use

d) Area R = Area Sector − Area of ΔABD

$= 19.6 - \frac{1}{2} \times 7 \times 3.5 \times \sin 0.8^c$

$= 19.6 - 8.78761\ldots$

$= 10.8123\ldots$

$= 10.8 \text{ cm}^2 \quad (3sf)$

radians mode on calculator

8.

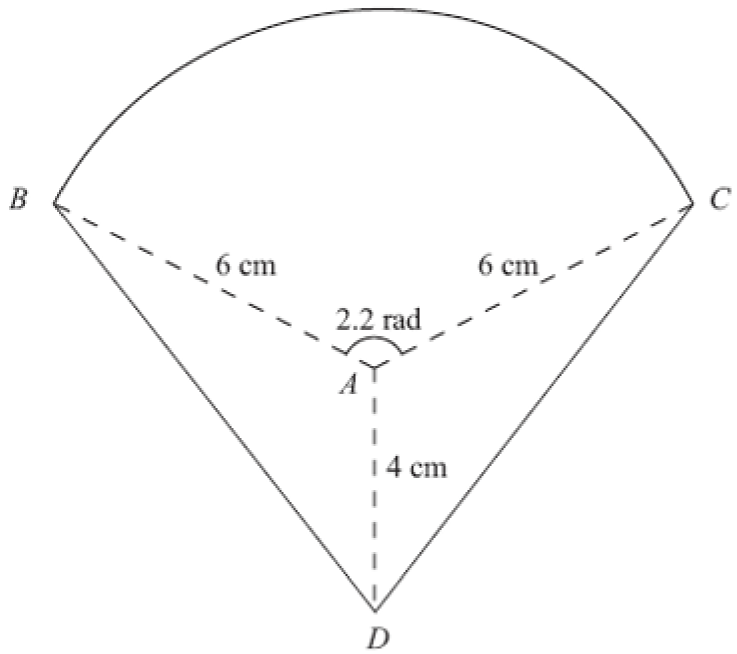


Figure 3

The shape BCD shown in Figure 3 is a design for a logo.

The straight lines DB and DC are equal in length. The curve BC is an arc of a circle with centre A and radius 6 cm. The size of $\angle BAC$ is 2.2 radians and $AD = 4$ cm.

Find

(*a*) the area of the sector BAC, in cm^2, **(2)**

(*b*) the size of $\angle DAC$, in radians to 3 significant figures, **(2)**

(*c*) the complete area of the logo design, to the nearest cm^2. **(4)**

a) Area sector BAC $= \frac{2.2}{2\cancel{\pi}_1} \times \cancel{\pi}^1 \times 6^2$

$= 39.6 cm^2$

b) As DB and DC are equal

Angle DAC = Angle DAB

$\angle DAC = \frac{(2\pi - 2.2)}{2} = 2.0415927$

$= 2.04$ radians (

7c) $\triangle$ BAD and $\triangle$ CAD are congruent

Area $\triangle BAD = \frac{1}{2} ab \sin C$

$= \frac{1}{2} \times 4 \times 6 \times \sin 2.0415927$

$= 10.694488 \text{ cm}^2$

Area of design $= 39.6 + 2(10.694488)$

$= 60.988976$

$= 61 \text{ cm}^2$ (nearest cm^2)

9.

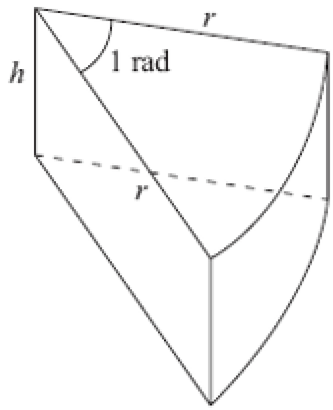


Figure 2

Figure 2 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(*a*) Show that the surface area of the box, S cm^2, is given by

$$S = r^2 + \frac{1800}{r}.$$ **(5)**

(*b*) Use calculus to find the value of r for which S is stationary. **(4)**

(*c*) Prove that this value of r gives a minimum value of S. **(2)**

(*d*) Find, to the nearest cm^2, this minimum value of S. **(2)**

Answers

a) Volume of cake = Area of sector × height.

$= \frac{1}{2} r^2 \theta \times h$

$\therefore \quad 300 = \frac{1}{2} r^2 (1) h \quad \Rightarrow \quad 300 = \frac{1}{2} r^2 h$

Solve for h: $\quad h = \frac{600}{r^2}$

Surface Area, $S = 2rh + lh + 2\left(\frac{1}{2} r^2 \theta\right)$

Recall $l = r\theta \Rightarrow l = r$ since $\theta = 1^c$

$S = 2rh + rh + r^2 = 3rh + r^2$

$\Rightarrow S = 3r\left(\frac{600}{r^2}\right) + r^2$

$\Rightarrow \underline{\underline{S = r^2 + \frac{1800}{r}}}$

b) $\frac{dS}{dr} = 2r - \frac{1800}{r^2}$

At Minimum: $\frac{dS}{dr} = 0$

$\therefore \quad 2r - \frac{1800}{r^2} = 0$

× all terms by r^2:

$2r^3 - 1800 = 0$

$r^3 = 900$

$\underline{\underline{r = 9.65}}$

c) $\dfrac{d^2S}{dr^2} = 2 + \dfrac{3600}{r^3}$

$\left.\dfrac{d^2S}{dr^2}\right|_{r=9.65} = 2 + \dfrac{3600}{(9.65)^3}$

$= 6$ which is >0.

$\Rightarrow$ S is a minimum when $r = 9.65$.

d) Sub $r = 9.65$

$S = (9.65)^2 + \dfrac{1800}{(9.65)}$

$S = 279.65$ cm^2

$\Rightarrow$ $\underline{\underline{S = 280 \text{ cm}^2}}$ to nearest cm^2.

10.

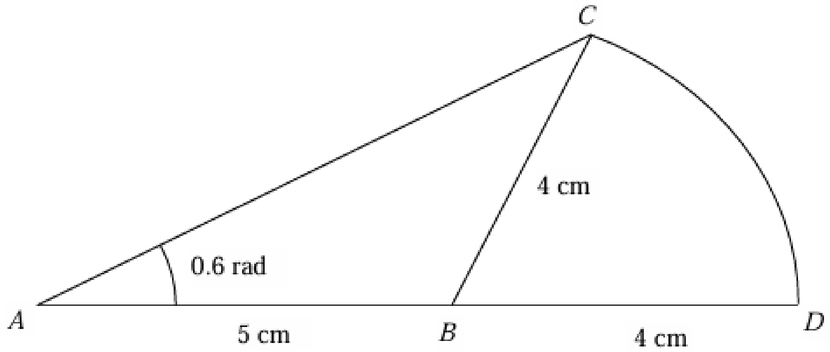


Figure 1

An emblem, as shown in Figure 1, consists of a triangle ABC joined to a sector CBD of a circle with radius 4 cm and centre B. The points A, B and D lie on a straight line with AB = 5 cm and BD = 4 cm. Angle BAC = 0.6 radians and AC is the longest side of the triangle ABC.

(*a*) Show that angle ABC = 1.76 radians, correct to three significant figures. **(4)**

(*b*) Find the area of the emblem. **(3)**

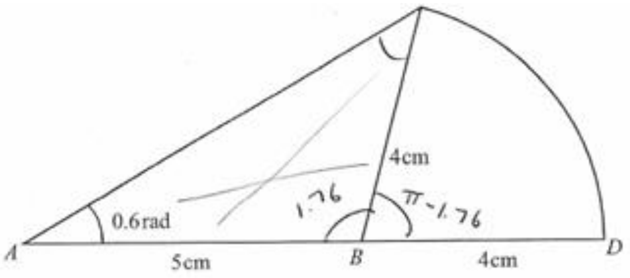


Figure 1

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(a) Show that angle $ABC = 1.76$ radians, correct to 3 significant figures. **(4)**

(b) Find the area of the emblem. **(3)**

a) Angle $A\hat{C}B$:

$$\frac{\sin(x)}{5} = \frac{\sin(0.6)}{4}$$

$$\sin(x) = \frac{5\sin(0.6)}{4}$$

$$\sin(x) = 0.7058\ldots$$

$$x = 0.7835561635$$

$$ABC = \pi - 0.6 - 0.7835561635$$

$$= 1.75803649$$

$$= 1.76^c \ (3sf)$$

b) Area of triangle $= \frac{1}{2}ab\sin C$

$$= \frac{1}{2}(4)(5)\sin(1.76)$$

$$= 9.825217144\ cm^2$$

11.

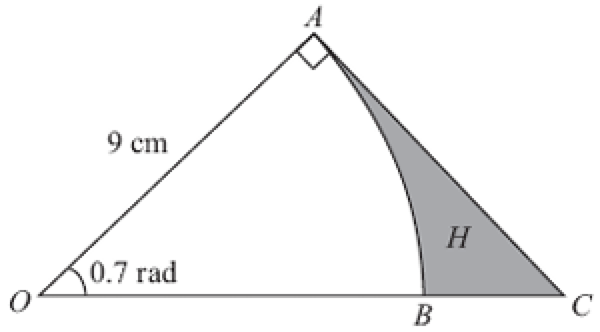


Figure 1

Figure 1 shows the sector OAB of a circle with centre O, radius 9 cm and angle 0.7 radians.

(a) Find the length of the arc AB. **(2)**

(b) Find the area of the sector OAB. **(2)**

The line AC shown in Figure 1 is perpendicular to OA, and OBC is a straight line.

(c) Find the length of AC, giving your answer to 2 decimal places. **(2)**

The region H is bounded by the arc AB and the lines AC and CB.

(d) Find the area of H, giving your answer to 2 decimal places. **(3)**

a) Arc Length $= \theta r$
$= 0.7 \times 9$
$= \underline{\underline{6.3\text{cm}}}$

b) Sector Area $= \frac{\theta}{2} r^2$
$= \frac{0.7}{2} \times 9^2$
$= 28.35\ \text{cm}^2$

c) $\tan(\theta) = \frac{o}{a}$
$\tan(0.7) = x/9$
$9 \tan(0.7) = x$
$x = 7.58$ cm (2d/p)

d) Area of triangle $= \frac{1}{2} \times 9 \times 7.58$
$= 34.1\ \text{cm}^2$ (3sf)

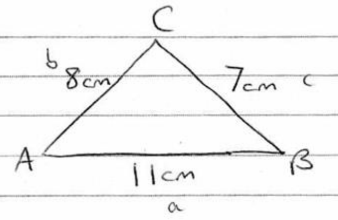
$34.1 - 28.35 = \underline{\underline{5.76\ \text{cm}^2}}$

12. In the triangle ABC, $AB = 11$ cm, $BC = 7$ cm and $CA = 8$ cm.

(*a*) Find the size of angle C, giving your answer in radians to 3 significant figures. **(3)**

(*b*) Find the area of triangle ABC, giving your answer in cm^2 to 3 significant figures. **(3)**

(3)



a) $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{(8)^2 + (7)^2 - (11)^2}{2(8)(\cancel{11}\,7)}$

$\cos A = -\dfrac{1}{14}$

$A = 1.6\cancel{7}4^c \ (3sf)$

b) $\text{Area} = \tfrac{1}{2} ab \sin C$

$= \tfrac{1}{2}(8)(7)\sin(1.6\cancel{4}4)$

$=$

$= 27.92848609$

$= 27.9\ \text{cm}^2\ (3sf)$

13.

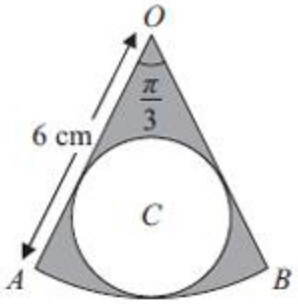


Figure 1

The shape shown in Figure 1 is a pattern for a pendant. It consists of a sector *OAB* of a circle centre *O*, of radius 6 cm, and angle $AOB = \frac{\pi}{3}$. The circle *C*, inside the sector, touches the two straight edges, *OA* and *OB*, and the arc *AB* as shown.

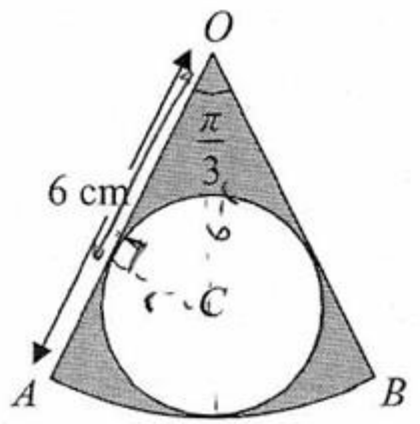
Find

(a) the area of the sector *OAB*, **(2)**

(b) the radius of the circle *C*. **(3)**

The region outside the circle *C* and inside the sector *OAB* is shown shaded in Figure 1.

(c) Find the area of the shaded region. **(2)**

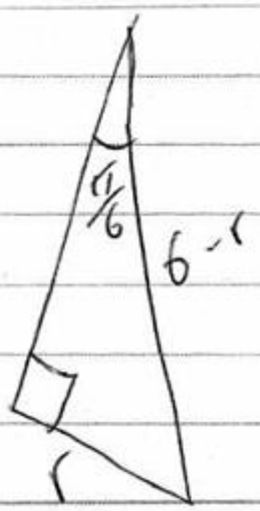


a/ sector area $= \frac{\theta}{2} r^2$

$= \frac{\pi/3}{2}(6)^2$

$= \underline{\underline{6\pi}}\ cm^2$

b/



$\sin\left(\frac{\pi}{6}\right) = \frac{r}{6-r}$

$\frac{1}{2} = \frac{r}{6-r}$

$6-r = 2r$

$6 = 3r$

$\underline{r = 2}\ cm$

c/ $6\pi - \pi(2)^2$

$6\pi - 4\pi$

$\underline{\underline{2\pi\ cm^2}}$