

- 1 Given that  $a > 0$ , determine which of these expressions is **not** equivalent to the others.

Circle your answer.

[1 mark]

$$-2 \log_{10} \left( \frac{1}{a} \right)$$

$$2 \log_{10} (a)$$

$$\log_{10} (a^2)$$

$$4 \log_{10} (\sqrt{a})$$

$$\log_{10} \left( \frac{1}{a} \right)^{-2} \\ = \log_{10} (a^2)$$

$$\log_{10} (a^2)$$

...

1

Which one of these functions is decreasing for all real values of  $x$ ?

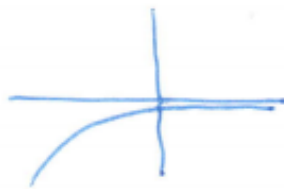
Circle your answer.

[1 mark]

$$f(x) = e^x$$



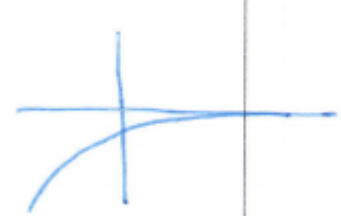
$$f(x) = -e^{1-x}$$



$$f(x) = -e^{x-1}$$



$$f(x) = -e^{-x}$$



4 The function  $f$  is defined by  $f(x) = e^{x-4}$ ,  $x \in \mathbb{R}$

Find  $f^{-1}(x)$  and state its domain.

[3 marks]

$$y = e^{x-4}$$

$$\ln y = x - 4$$

$$\ln y + 4 = x$$

$$f^{-1}(x) = \ln x + 4$$

domain  $x > 0$

(as  $\ln x$  undefined for  
 $x \leq 0$ )

7 (a) Given that  $\log_a y = 2\log_a 7 + \log_a 4 + \frac{1}{2}$ , find  $y$  in terms of  $a$ .

[4 marks]

$$\log_a y - \log_a 7^2 - \log_a 4 = \frac{1}{2}$$

$$\log_a \frac{y}{7^2 \times 4} = \frac{1}{2}$$

$$\frac{y}{196} = a^{\frac{1}{2}}$$

$$y = 196 a^{\frac{1}{2}}$$

7 (b) When asked to solve the equation

$$2 \log_a x = \log_a 9 - \log_a 4$$

a student gives the following solution:

$$2 \log_a x = \log_a 9 - \log_a 4$$

$$\Rightarrow 2 \log_a x = \log_a \frac{9}{4}$$

$$\Rightarrow \log_a x^2 = \log_a \frac{9}{4}$$

$$\Rightarrow x^2 = \frac{9}{4}$$

$$\therefore x = \frac{3}{2} \text{ or } -\frac{3}{2}$$

Explain what is wrong with the student's solution.

[1 mark]

Cannot evaluate  $\log_a(-\frac{3}{2})$

so reject solution

$$x = -\frac{3}{2}$$

- 10 A scientist is researching the effects of caffeine. She models the mass of caffeine in the body using

$$m = m_0 e^{-kt}$$

where  $m_0$  milligrams is the initial mass of caffeine in the body and  $m$  milligrams is the mass of caffeine in the body after  $t$  hours.

On average, it takes 5.7 hours for the mass of caffeine in the body to halve.

One cup of strong coffee contains 200 mg of caffeine.

- 10 (a) The scientist drinks two strong cups of coffee at 8 am. Use the model to estimate the mass of caffeine in the scientist's body at midday.

[4 marks]

$$\text{at } t = 0, m = 400$$

$$400 = m_0 e^{-0} \Rightarrow m_0 = 400$$

$$\text{at } t = 5.7, m = 200 \text{ (halved)}$$

$$200 = 400 e^{-5.7k}$$

$$\frac{200}{400} = e^{-5.7k}$$

$$\ln\left(\frac{1}{2}\right) = -5.7k$$

$$k = \frac{\ln\left(\frac{1}{2}\right)}{-5.7} = 0.1216047685$$

$$\text{at midday, } t = 4$$

$$m = 400 e^{-0.1216047685 \times 4}$$

$$m = 245.9296 \text{ mg}$$

10 (b) The scientist wants the mass of caffeine in her body to stay below 480 mg

Use the model to find the earliest time that she could drink another cup of strong coffee.

Give your answer to the nearest minute.

[3 marks]

Stay below 480  $\rightarrow$  drop to 280 mg

then can drink 1 cup

$$280 = 400e^{-0.1216047685t}$$

$$\ln\left(\frac{280}{400}\right) = -0.1216047685t$$

$$t = \frac{\ln\left(\frac{280}{400}\right)}{-0.1216047685}$$

$$t = 2.829682623 \text{ hours}$$

$$\text{time} = 2 \text{ hrs} + 0.829682623 \times 60$$

(using  $k = -0.1216047685$ ) = 2 hrs + 50 mins

(mark scheme rounded to  $-0.1216$  for 10:56am) = 10:50 am

10 (c) State a reason why the mass of caffeine remaining in the scientist's body predicted by the model may not be accurate.

[1 mark]

Different people will eliminate caffeine from the body at different rates

8 Theresa bought a house on 2 January 1970 for £8000.

The house was valued by a local estate agent on the same date every 10 years up to 2010.

The valuations are shown in the following table.

Year	1970	1980	1990	2000	2010
Valuation price	£8 000	£19 000	£36 000	£82 000	£205 000

The valuation price of the house can be modelled by the equation

$$V = pq^t$$

where  $V$  pounds is the valuation price  $t$  years after 2 January 1970 and  $p$  and  $q$  are constants.

8 (a) Show that  $V = pq^t$  can be written as  $\log_{10} V = \log_{10} p + t \log_{10} q$

[2 marks]

$$V = p q^t$$

$$\log_{10} V = \log_{10} p q^t$$

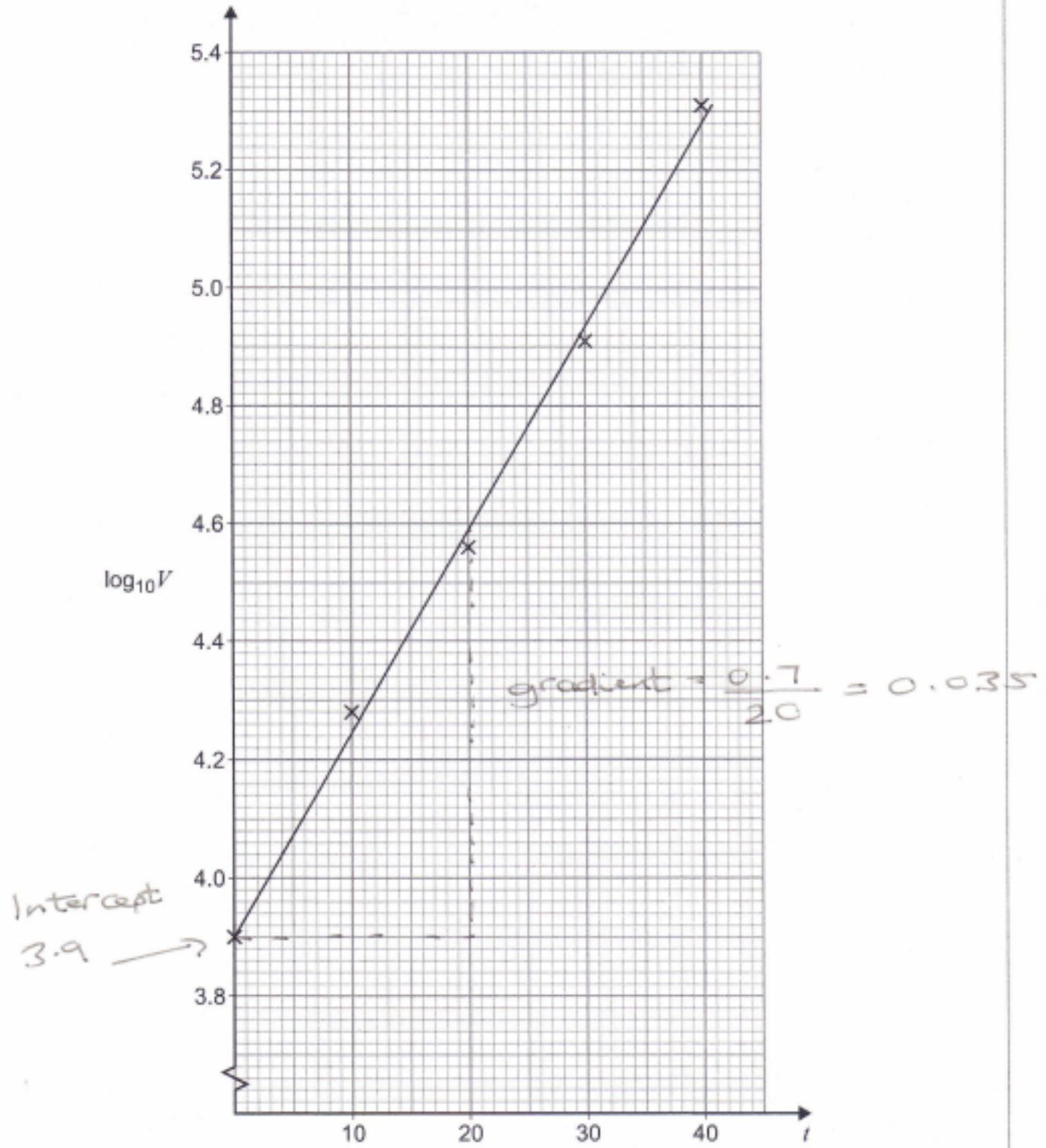
$$\log_{10} V = \log_{10} p + \log_{10} q^t$$

$$\log_{10} V = \log_{10} p + t \log_{10} q$$



8 (b) The values in the table of  $\log_{10} V$  against  $t$  have been plotted and a line of best fit has been drawn on the graph below.

$t$	0	10	20	30	40
$\log_{10} V$	3.90	4.28	4.56	4.91	5.31



Question 8 continues on the next page



- 8 A student is conducting an experiment in a laboratory to investigate how quickly liquids cool to room temperature.

A beaker containing a hot liquid at an initial temperature of  $75^{\circ}\text{C}$  cools so that the temperature,  $\theta^{\circ}\text{C}$ , of the liquid at time  $t$  minutes can be modelled by the equation

$$\theta = 5(4 + \lambda e^{-kt})$$

where  $\lambda$  and  $k$  are constants.

After 2 minutes the temperature falls to  $68^{\circ}\text{C}$ .

- 8 (a) Find the temperature of the liquid after 15 minutes.

Give your answer to three significant figures.

[7 marks]

$$\text{at } t=0, \theta = 75$$

$$75 = 5(4 + \lambda e^{-0})$$

$$\frac{75}{5} = 4 + \lambda$$

$$\lambda = 15 - 4 = 11$$

$$\text{at } t=2, \theta = 68$$

$$68 = 5(4 + 11e^{-2k})$$

$$\frac{68}{5} = 4 + 11e^{-2k}$$

$$\frac{68}{5} - 4 = e^{-2k}$$

$$11$$

$$\frac{4.8}{55} = e^{-2k}$$

$$\ln \frac{4.8}{55} = -2k$$

$$k = -\frac{1}{2} \ln \frac{4.8}{55} = 0.068066$$

$$\text{at } t=15, \theta = 5(4 + 11e^{-15 \times 0.068066})$$

$$\theta = 39.813$$

$$\theta = 39.8^{\circ}\text{C} \quad (3\text{sf})$$

8 (b) (i) Find the room temperature of the laboratory, giving a reason for your answer.

[2 marks]

as  $t \rightarrow \infty$  the liquid will  
approach room temperature

$$\theta = 5(4 + 11e^{-\lambda t})$$

$\uparrow 0$

$$\theta = 5 \times 4 = 20^\circ\text{C}$$

8 (b) (ii) Find the time taken in minutes for the liquid to cool to  $1^\circ\text{C}$  above the room temperature of the laboratory.

[2 marks]

$$21 = 5(4 + 11e^{-0.068066t})$$

$\uparrow 21^\circ\text{C}$

$$\frac{21 - 4}{5} = e^{-0.068066t}$$

$$\left(\frac{1}{55}\right) = e^{-0.068066t}$$

$$\ln\left(\frac{1}{55}\right) = t$$

$$-0.068066t = 58.87 \text{ minutes (4sf)}$$

8 (c) Explain why the model might need to be changed if the experiment was conducted in a different place.

[1 mark]

Different place may have a  
different room temperature  
so it would cool at a different  
rate.