Friction

Model Answers 1

Time Allowed: **47 minutes**

Score: **/39**

Percentage: **/100**

Grade Boundaries:

A*	A	B	C	D	E	U
>85%	77.5%	70%	62.5%	57.5%	45%	<45%

Model Answer Key

Red = Answer

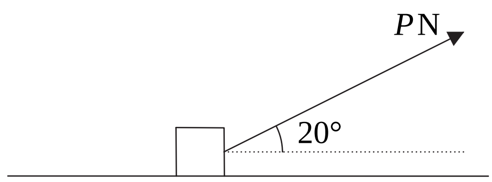
- This is what you need to write to get the mark
- Each bullet point represents M Y l·ŊJ l

Blue = Explanation

- This is here to help you understand the answer
- You DON'T need to write this to get the marks

1.

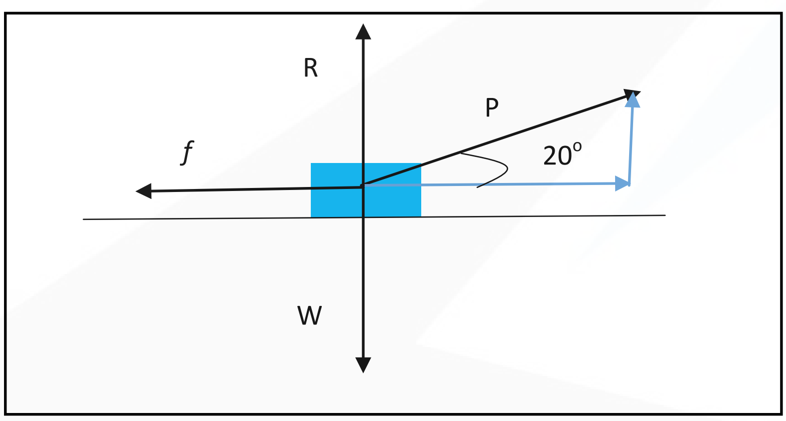
Figure 3



A box of mass 30 kg is being pulled along rough horizontal ground at a constant speed using a rope. The rope makes an angle of 20° with the ground, as shown in Figure 3. The coefficient of friction between the box and the ground is 0.4. The box is modelled as a particle and the rope as a light, inextensible string. The tension in the rope is P newtons.

(a) Find the value of P. **(8)**

The block is resting on a surface so it feels a normal reaction force (R) perpendicular to the surface. Black arrows are forces, blue are components.



$P\cos 20 - f = 0$ Horizontal forces are in equilibrium, so there is no acceleration.

Hence using F=ma tells us the sum of forces is zero

$f = \mu R$ Equation for frictional limit

$P\cos 20 = \mu R$ Sub in equation for frictional limit

$R + P\sin 20 - W = 0$ The vertical forces are also in equilibrium and so add up to zero for the same reasons as the horizontal ones.

$R = W - P\sin 20$ Make R subject so it can be subbed into horizontal equation

$\mu(W - P\sin 20) = P\cos 20$

$\mu W = \mu P\sin 20 + P\cos 20$ Expand and move terms containing P to one side

$\mu W = P(\mu\sin 20 + \cos 20)$ Factorise P out on the right hand side

$$P = \frac{\mu W}{\mu \sin 20 + \cos 20}$$

$$= \frac{0.4 * 30 * 9.8}{0.4 \sin 20 + \cos 20}$$

$$= \mathbf{109\ N}$$

The tension in the rope is now increased to 150 N.

(b) Find the acceleration of the box. **(6)**

Diagram is same as part a, now we also have the knowledge of P=150N

$30g = R + 150 \sin 20$Equation for vertical forces is the same as before except p=150 and W=30g are subbed in

$150 \cos 20 - 0.4R = 30a$ Compare horizontal forces using F=ma, the same forces are present as in part a, however they now cause an acceleration

$$a = \frac{150 \cos 20 - 12g + 60 \sin 20}{30}$$

$$= \mathbf{1.46\ ms^{-2}}$$

(Total 14 marks)

2.

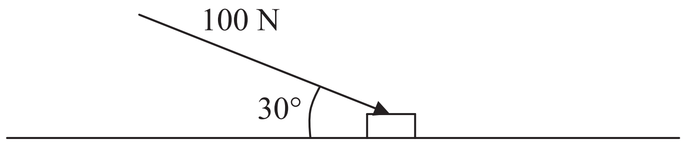


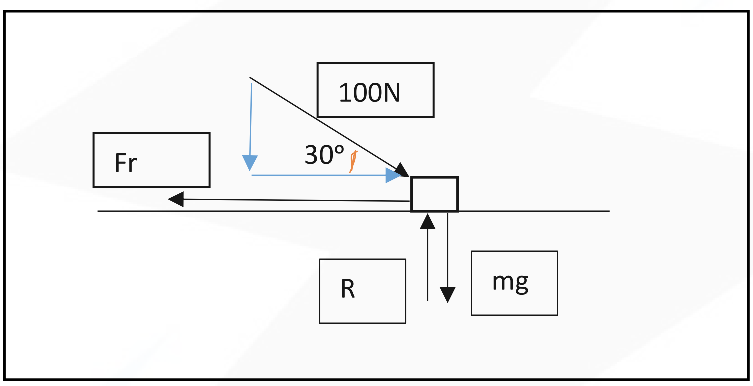
Figure 1

A small box is pushed along a floor. The floor is modelled as a rough horizontal plane and the box is modelled as a particle. The coefficient of friction between the box and the floor is $\frac{1}{2}$. The box is pushed by a force of magnitude 100 N which acts at an angle of 30° with the floor, as shown in Figure 1.

Given that the box moves with constant speed, find the mass of the box. **(7)**

If the box moves with constant speed, it experiences **no net force**. This is because of Newton's second law: $\sum F = ma$, where acceleration is zero, so the forces must balance.

The horizontal forces on the box are the horizontal component of the push, and the friction force in the opposite direction.



The horizontal component of the pushing force is 100 x cos(30)

As the box is moving, it is in limiting friction. Therefore friction (Fr) is μ x R, where R is the normal reaction force.

So $Fr = 100\cos 30 = \mu R$

The normal reaction is due to the weight of the box, mg, and the vertical component of the pushing force, $R = mg + 100\sin 30$.

$$100\cos 30 = \mu\,(mg + 100\sin 30)$$

$100\cos30 = 0.5\,(mg + 100\sin30)$	substitute in μ
$0.5mg = 100\cos30 - 50\sin30$	rearrange for 0.5mg
$m = \dfrac{100\cos30 - 50\sin30}{0.5g}$	rearrange for m
m = 12.6 kg	

(Total 7 marks)

3.

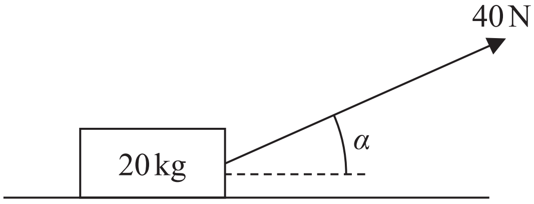


Figure 1

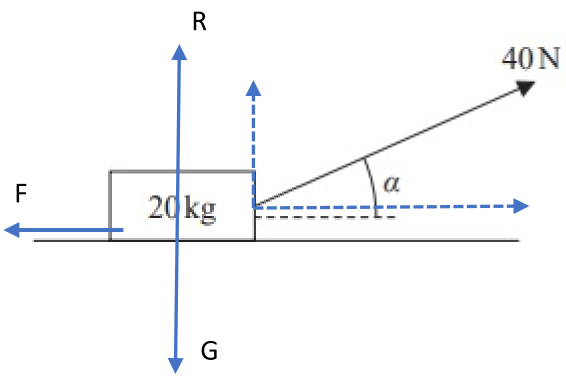
A wooden crate of mass 20 kg is pulled in a straight line along a rough horizontal floor using a handle attached to the crate.

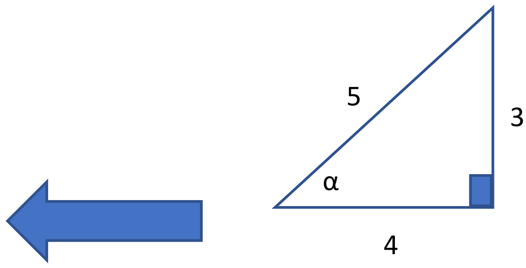
The handle is inclined at an angle α to the floor, as shown in Figure 1, where $\tan \alpha = \frac{3}{4}$

The tension in the handle is 40 N.
The coefficient of friction between the crate and the floor is 0.14
The crate is modelled as a particle and the handle is modelled as a light rod.

Using the model,

(a) find the acceleration of the crate. **(6)**





$$tan\alpha = \frac{3}{4}$$

Using Pythagoras' Theorem, hypotenuse = 5, thus:

$sin\alpha = \frac{3}{5}$ and $cos\alpha = \frac{4}{5}$

We can apply Newton's Second Law of Motion:

$\sum F$ = ma

We need to resolve both vertically and horizontally since we have forces in with both directions. We consider the force equal to 40 N according to its vertical and horizontal components.

Vertically, we consider up as positive.

R + 40 sin α – G = 0

R + 40 sin α = 20g

$$sin\alpha = \frac{3}{5} = 0.6$$

R + 40 x 0.6 = 20g

R = 172 N

Horizontally, we consider right as positive.

40 cos α – F = ma

$$cos\alpha = \frac{4}{5} = 0.8$$

F = 40 x 0.8 - 20a

F = μR

F = 172 x 0.14

F = 24.08 N

24.08 = 32 – 20a

a = 0.396 ms^{-2}

The crate is now pushed along the same floor using the handle. The handle is again inclined at the same angle α to the floor, and the thrust in the handle is 40 N as shown in Figure 2 below.

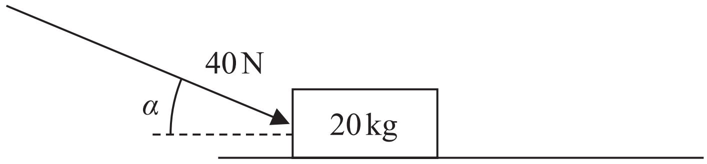
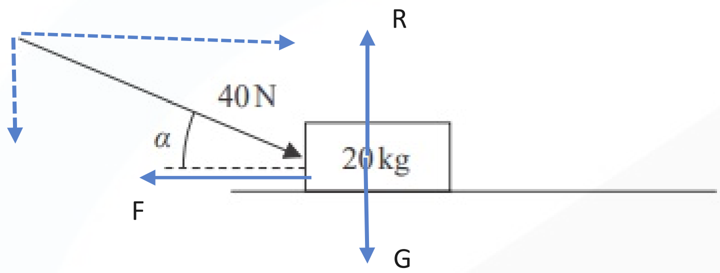


Figure 2

(b) Explain briefly why the acceleration of the crate would now be less than the acceleration of the crate found in part (a).

(2)



In this situation, if we solve it vertically in similar fashion to part a) we obtain:

$R - G - 40 \sin \alpha = 0$

$R = 20g + 40 \sin \alpha$

In this case, R will be greater than the R obtained at point a).

$F = \mu R$

Therefore, if R increases than the force of friction F will increase as well.

If we solve it horizontally, similar to point a) we obtain:

$40 \cos \alpha - F = 20a$

F in this equation is bigger, therefore, a becomes smaller.

(Total 8 marks)

4.

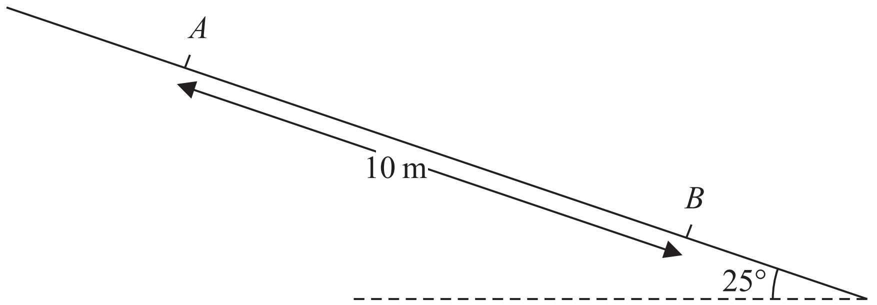
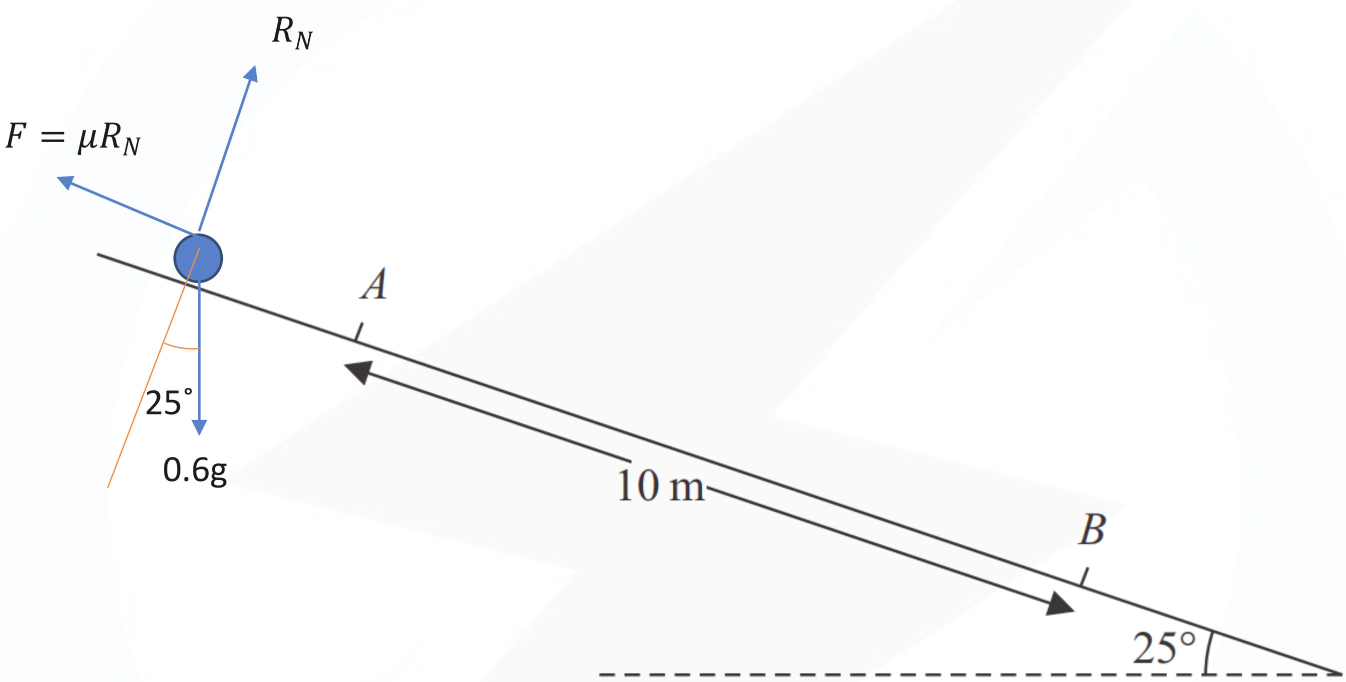


Figure 3

A particle P of mass 0.6 kg slides with constant acceleration down a line of greatest slope of a rough plane, which is inclined at 25° to the horizontal. The particle passes through two points A and B, where AB = 10 m, as shown in Figure 3. The speed of P at A is 2 m s^{-1}. The particle P takes 3.5 s to move from A to B. Find

(a) the speed of P at B,

(3)



Using SUVAT we have

$$s = \frac{1}{2}(u+v)t$$

$$\rightarrow 10 = \frac{1}{2}(2+v)3.5$$

$$\rightarrow v = \frac{40}{7} - 2$$

$$= \frac{26}{7} \approx 3.71ms^{-1}$$

(b) the acceleration of P,

(2)

Using SUVAT

$$\boldsymbol{v = u + at}$$

$$\rightarrow a = \frac{v-u}{t}$$

$$\rightarrow a = \frac{1}{3.5}\left(\frac{26}{7} - 2\right)$$

$$= \boldsymbol{\frac{24}{49} \approx 0.490ms^{-2}}$$

(c) the coefficient of friction between P and the plane.

(5)

Using Newton's 2nd law we have

$$\boldsymbol{F = ma}$$

$$\rightarrow 0.6g \sin 25 - \mu R_N = 0.6a$$

$$\rightarrow \mu = \frac{0.6\left(g \sin 25 - \frac{24}{49}\right)}{R_N} \quad (1)$$

Resolving forces perpendicular to the plane we get

$$0.6g \cos 25 = R_N$$

Substituting this into (1) we get

$$\boldsymbol{\mu = 0.41}$$

(Total 10 marks)