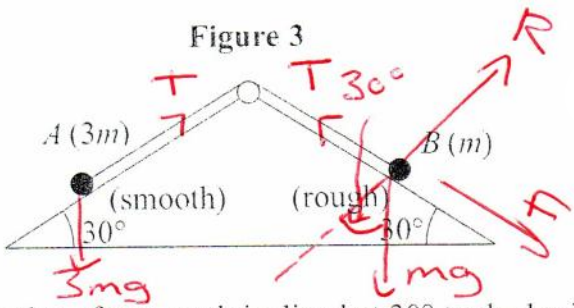
7.

Figure 3



A fixed wedge has two plane faces, each inclined at 30° to the horizontal. Two particles A and B, of mass $3m$ and m respectively, are attached to the ends of a light inextensible string. Each particle moves on one of the plane faces of the wedge. The string passes over a small smooth light pulley fixed at the top of the wedge. The face on which A moves is smooth. The face on which B moves is rough. The coefficient of friction between B and this face is μ. Particle A is held at rest with the string taut. The string lies in the same vertical plane as lines of greatest slope on each plane face of the wedge, as shown in Figure 3.

The particles are released from rest and start to move. Particle A moves downwards and B moves upwards. The accelerations of A and B each have magnitude $\frac{1}{10}g$.

(a) By considering the motion of A, find, in terms of m and g, the tension in the string. **(3)**

(b) By considering the motion of B, find the value of μ. **(8)**

(c) Find the resultant force exerted by the string on the pulley, giving its magnitude and direction. **(3)**

a) Equation of motion for A

R ($\swarrow$) parallel to plane

$3m \times a = 3mg \sin 30° - T$

$a = \frac{g}{10}$

$3m \times \frac{g}{10} = 3mg \sin 30° - T$

$T = 3mg \sin 30° - \frac{3mg}{10}$

$T = 1.2mg$ N

Leave blank

Question 7 continued

7b) R (↗) perpendicular to plane

$R - mg\cos 30° = 0$ (no motion perpendicular to plane)

$R = mg\cos 30°$

R (↖) parallel to plane

$m \times \frac{1}{10} g = T - F - mg\sin 30°$

$F = T - mg\sin 30° - \frac{mg}{10}$

$F = 1.2mg - 0.5mg - 0.1mg$

$F = 0.6mg$ N

But $F = \mu R$

$\therefore \mu = \frac{F}{R}$

$\mu = \frac{0.6\cancel{mg}}{\cancel{mg}\cos 30°} = 0.6928203$

$\mu = 0.693$ (3 sf)

Question 7 continued

7c)

Force exerted on pulley by string

$= T\cos 60° + T\cos 60°$

$= 2T\cos 60°$

$= \cancel{2} \times \frac{6}{5}mg \times \frac{1}{\cancel{2}}$

Force $= \frac{6}{5}mg$

Direction is vertically downwards

Q7

(Total 14 marks)

TOTAL FOR PAPER: 75 MARKS

END

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7.

Figure 4

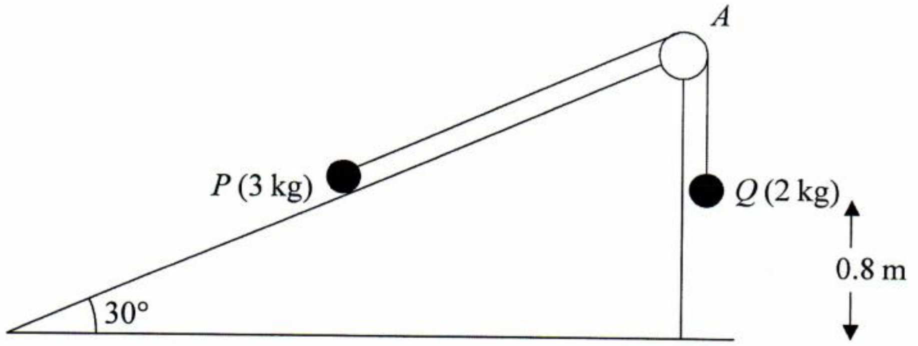


Figure 4 shows two particles P and Q, of mass 3 kg and 2 kg respectively, connected by a light inextensible string. Initially P is held at rest on a fixed smooth plane inclined at 30° to the horizontal. The string passes over a small smooth light pulley A fixed at the top of the plane. The part of the string from P to A is parallel to a line of greatest slope of the plane. The particle Q hangs freely below A. The system is released from rest with the string taut.

(a) Write down an equation of motion for P and an equation of motion for Q.

(4)

(b) Hence show that the acceleration of Q is 0.98 m s^{-2}.

(2)

(c) Find the tension in the string.

(2)

(d) State where in your calculations you have used the information that the string is inextensible.

(1)

On release, Q is at a height of 0.8 m above the ground. When Q reaches the ground, it is brought to rest immediately by the impact with the ground and does not rebound. The initial distance of P from A is such that in the subsequent motion P does not reach A. Find

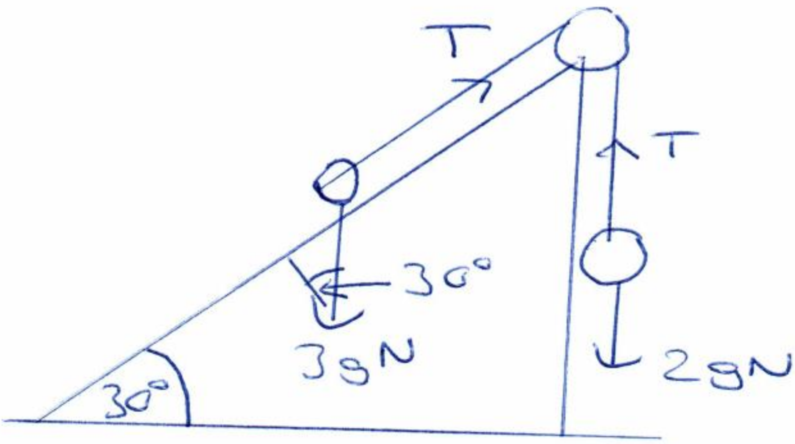
(e) the speed of Q as it reaches the ground,

(2)

(f) the time between the instant when Q reaches the ground and the instant when the string becomes taut again.

(5)

7 a)



Equation of Motion for Q

$(\downarrow)^{+ve}$ $2g - T = 2a$ ①

Equation of Motion for P

$(\nearrow)^{+ve}$ along plane $T - 3g\sin 30° = 3a$ ②

b) Using equation ① for Q

$2g - T = 2a$

$\therefore T = 2g - 2a$

using this in ②

$2g - 2a - 3g\sin 30° = 3a$

$\therefore 2g - 3g\sin 30° = 5a$

$\therefore a = \dfrac{2 \times 9.8 - 3 \times 9.8 \times \sin 30°}{5}$

$\therefore a = 0.98\ \text{ms}^{-2}$

c) using ①

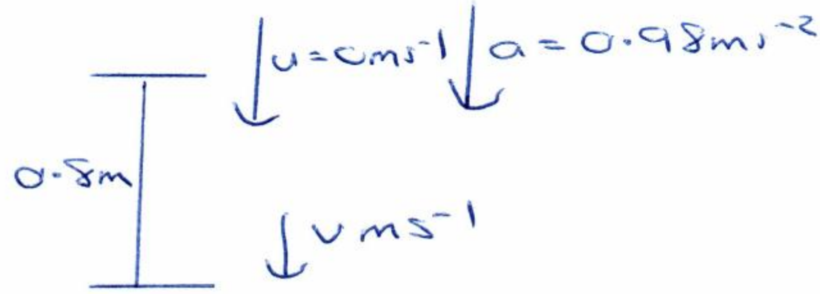
$T = 2g - 2a$

$= 2 \times 9.8 - 2 \times 0.98 = 17.64$

$\therefore T = 17.6\ \text{N}$ (3 sf)

d) The magnitudes of the accelerations of P and Q are equal

7 e)



$\downarrow$ +ve $\quad s = 0.8m, u = 0ms^{-1}, a = 0.98ms^{-2}$

$v = ?$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times 0.98 \times 0.8$

$v^2 = 1.568$

$v = \sqrt{1.568} = 1.2521981$

$= 1.25 ms^{-1}$ (3 sf)

f)

We need to find the time for P to go up slope and back to its original position (s=0, displacement from start to end of motion is zero).

* Also, T = 0 as string goes slack during the motion up/down until it becomes taut again

First, equation of Motion for P to find its acceleration.

$T - 3g \sin 30° = 3a$

$0 - 3g \sin 30° = 3a$

$\therefore a = -\dfrac{3 \times 9.8 \times \sin 30°}{3} = -4.9 ms^{-2}$

7f (continued)

using $s = ut + \frac{1}{2}at^2$

+ve ↑ $s = 0$, $u = \sqrt{1.568}\ ms^{-1}$, $a = -4.9\ ms^{-2}$, $t = ?$

$0 = \sqrt{1.568}\ t + \frac{1}{2} \times -4.9 \times t^2$

$0 = t\left(\sqrt{1.568} - 2.45t\right)$

Either $t = 0$ (start) or $t = \frac{\sqrt{1.568}}{2.45}$

$t = 0.5111012$

So $t = 0.511$ seconds (3 sf) when string becomes taut again.

7.

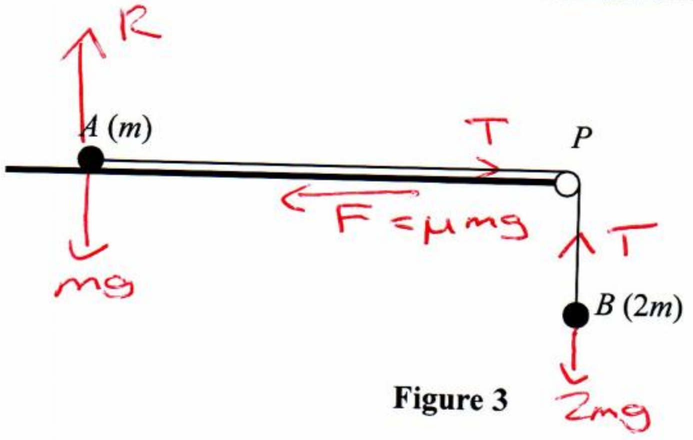


Figure 3

Two particles A and B, of mass m and $2m$ respectively, are attached to the ends of a light inextensible string. The particle A lies on a rough horizontal table. The string passes over a small smooth pulley P fixed on the edge of the table. The particle B hangs freely below the pulley, as shown in Figure 3. The coefficient of friction between A and the table is μ. The particles are released from rest with the string taut. Immediately after release, the magnitude of the acceleration of A and B is $\frac{4}{9}g$. By writing down separate equations of motion for A and B,

(a) find the tension in the string immediately after the particles begin to move, **(3)**

(b) show that $\mu = \frac{2}{3}$. **(5)**

When B has fallen a distance h, it hits the ground and does not rebound. Particle A is then a distance $\frac{1}{3}h$ from P.

(c) Find the speed of A as it reaches P. **(6)**

(d) State how you have used the information that the string is light. **(1)**

a) For B, R (↓) $2m \times a = 2mg - T$

$2m \times \frac{4}{9}g = 2mg - T$

$T = 2mg - \frac{8}{9}mg$

$T = \frac{10}{9}mg$ (1)

b) For A R (→)

$m \times a = T - F$

$m \times \frac{4}{9}g = \frac{10}{9}mg - F$ (2)

R (↑) for A $R - mg = 0$

$R = mg$

Also $F = \mu R = \mu mg$ (3)

Leave blank

7b) continued

put $F = \mu m g$ from (3) in (2)

$$\cancel{m} \times \frac{4}{9}\cancel{g} = \frac{10}{9}\cancel{mg} - \mu \cancel{mg}$$

$$\mu = \frac{10}{9} - \frac{4}{9} = \frac{6}{9} = \frac{2}{3}$$

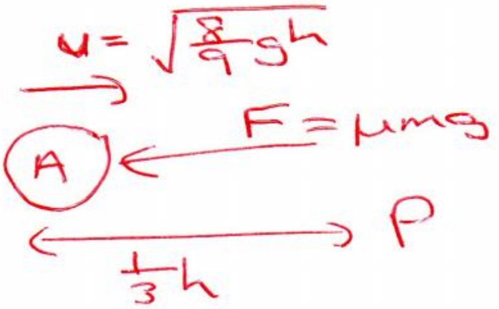
7c) When B hits the ground consider the motion of A

$s = h\,m,\ u = 0,\ v = ?,\ a = \frac{4}{9}g,\ t = ?$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times \frac{4}{9} \times 9.8 \times h$$

$$v^2 = \frac{8}{9}gh$$



Equation of motion for A is

R ($\rightarrow$)

$$\cancel{m} \times a = -\mu \cancel{m} g$$

$$a = -\mu g$$

$$a = -\frac{2}{3}g$$

$$\boxed{v^2 = u^2 + 2as}$$

To get speed at P for A

$s = \frac{1}{3}h,\ u = \sqrt{\frac{8}{9}gh},\ v = ?,\ a = -\frac{2}{3}g,\ t = ?$

$$v^2 = \left(\sqrt{\frac{8}{9}gh}\right)^2 + 2\left(-\frac{2}{3}g\right) \times \frac{1}{3}h$$

$$v^2 = \frac{8}{9}gh \quad \frac{4}{9}gh$$

$$v^2 = \frac{4}{9}gh = \frac{4}{9}gh$$

$$v = \sqrt{\frac{4}{9}gh} = \frac{2}{3}\sqrt{gh}\ \text{m s}^{-1}$$

d) Same Tension on A and B

7 a) continued

Substitute $T = 15g - 15a$ in (i)

$$5a = (15g - 15a) - 5g \times \frac{3}{5}$$

$$15a + 5a = 147 - 29.4$$

$$20a = 117.6$$

(i) $a = 5.88\ ms^{-2}$

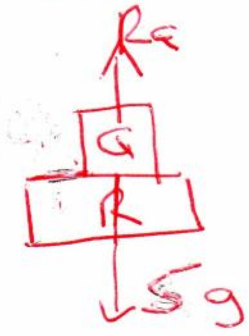
acceleration of the scale pan is $5.88\ ms^{-2}$

(ii) substitute $a = 5.88\ ms^{-2}$ in (i)

$$5 \times 5.88 = T - \left(5 \times 9.8 \times \frac{3}{5}\right)$$

$$T = 29.4 + 29.4$$

$$T = 58.8\ N$$

b) 

$\downarrow a = 5.88\ ms^{-2}$

Force exerted by Q on R

For Q only $R(\uparrow)$

$$R_Q - 5g = 5 \times -5.88$$

$$R_Q = (5 \times 9.8) - 5(5.88)$$

$$R_Q = 19.6\ N$$

7.

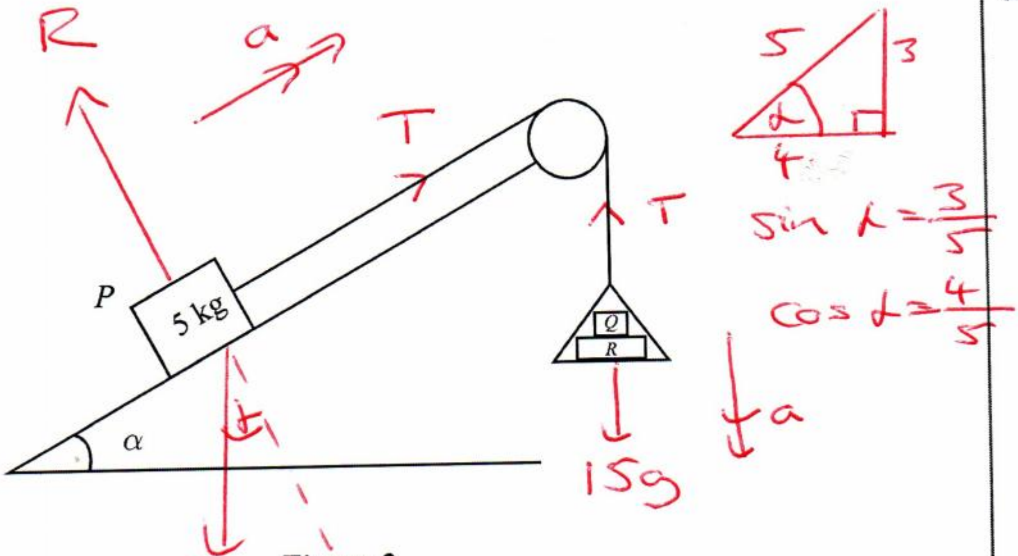


Figure 3

One end of a light inextensible string is attached to a block P of mass 5 kg. The block P is held at rest on a smooth fixed plane which is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{3}{5}$. The string lies along a line of greatest slope of the plane and passes over a smooth light pulley which is fixed at the top of the plane. The other end of the string is attached to a light scale pan which carries two blocks Q and R, with block Q on top of block R, as shown in Figure 3. The mass of block Q is 5 kg and the mass of block R is 10 kg. The scale pan hangs at rest and the system is released from rest. By modelling the blocks as particles, ignoring air resistance and assuming the motion is uninterrupted, find

(a) (i) the acceleration of the scale pan,

(ii) the tension in the string,

(8)

(b) the magnitude of the force exerted on block Q by block R,

(3)

(c) the magnitude of the force exerted on the pulley by the string.

(5)

a) Eqn of motion for P (parallel to plane)

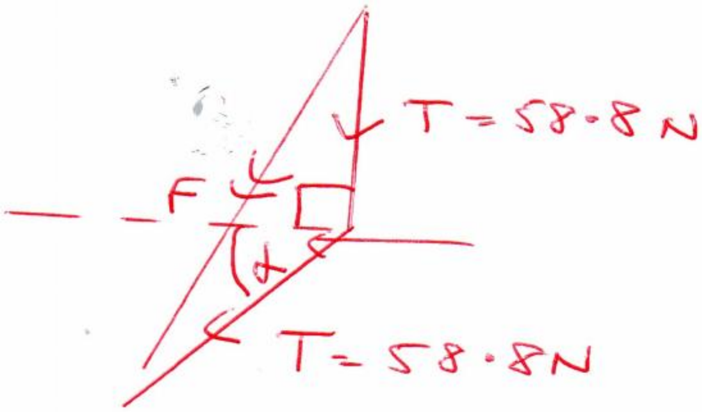
$5 \times a = T - 5g\sin\alpha$ (1)

Eqn of motion for Q+R only

$15 \times a = 15g - T$ (2)

$\therefore T = 15g - 15a$

7c)



$\sin\alpha = \frac{3}{5}$

$\alpha = \sin^{-1}\frac{3}{5}$

$\alpha = 36{\cdot}869°$

$\therefore$ Angle between T and T

is $90 + \alpha = 126{\cdot}8699°$

Using cosine rule to get resultant F

$F^2 = T^2 + T^2 - 2 \times T \times T \times \cos 126{\cdot}8699°$

$F^2 = 11063{\cdot}3808$

$F = 105{\cdot}18464$

$F = 105$ N (3 sf) is the force exerted by the pulley on the string.

6.

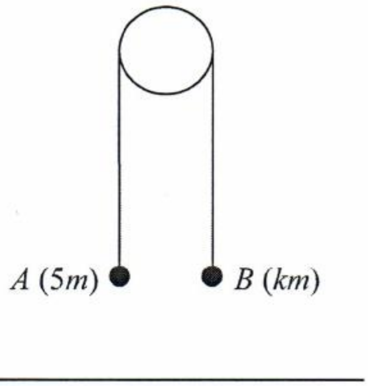


Figure 4

Two particles A and B have masses $5m$ and km respectively, where $k < 5$. The particles are connected by a light inextensible string which passes over a smooth light fixed pulley. The system is held at rest with the string taut, the hanging parts of the string vertical and with A and B at the same height above a horizontal plane, as shown in Figure 4. The system is released from rest. After release, A descends with acceleration $\frac{1}{4}g$.

(a) Show that the tension in the string as A descends is $\frac{15}{4}mg$. **(3)**

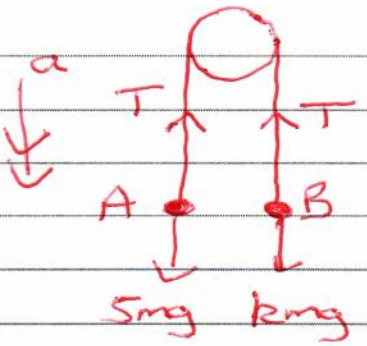
(b) Find the value of k. **(3)**

(c) State how you have used the information that the pulley is smooth. **(1)**

After descending for 1.2 s, the particle A reaches the plane. It is immediately brought to rest by the impact with the plane. The initial distance between B and the pulley is such that, in the subsequent motion, B does not reach the pulley.

(d) Find the greatest height reached by B above the plane. **(7)**

a)



Equation of motion for A

R (↓)

$5m \times a = 5mg - T$

$T = 5mg - 5m \times \frac{1}{4}g$

$T = \frac{20}{4}mg - \frac{5}{4}mg$

$T = \frac{15}{4}mg$

(as required)

6b) Equation of motion for B (moving upwards)

R (↑)

$$km \times a = T - kmg$$

$$km \times \frac{1}{4}g = \frac{15}{4}mg - kmg$$

$$\frac{1}{4}k\cancel{m}\cancel{g} + k\cancel{m}\cancel{g} = \frac{15}{4}\cancel{m}\cancel{g}$$

$$\frac{5}{4}k = \frac{15}{4}$$

$$k = 3$$

c) The tensions in the two parts of the string are the same.

d) Find speed on reaching ground first

$s = ?$ $u = 0$, $v = ?$ $a = \frac{1}{4}g$, $t = 1.2$ secs

R (↓) $v = u + at$

$$v = 0 + \frac{1}{4} \times 9.8 \times 1.2$$

$v = 2.94\ ms^{-1}$ on reaching ground

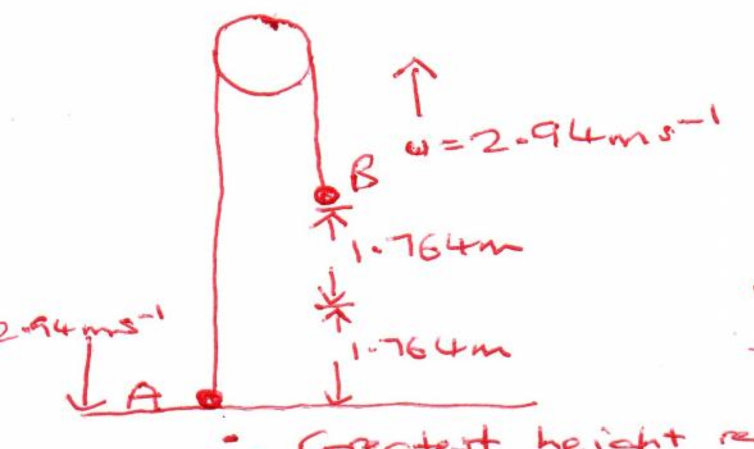
Distance between A's starting point and the ground

$s = ?$ $u = 0$, $t = 1.2$ secs, $a = \frac{1}{4}g$

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2} \times \frac{1}{4} \times 9.8 \times 1.2^2$$

$$s = 1.764\ m$$



R (↑) $u = 2.94 ms^{-1}$

$a = -9.8 ms^{-2}$

$v = 0 ms^{-1}$

$s = ?$

Gets to highest point when $v = 0\ ms^{-1}$

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - (2.94)^2}{2 \times -9.8}$$

$$s = 0.441\ m$$

∴ Greatest height reached $= 0.441 + 1.764 + 1.764$

$= 3.963\ m$

M1 JAN 2011

7a) continued

(3) gives $R = 3 \times g \times \frac{12}{13}$

Friction $F = \mu R$

$$F = \frac{2}{3} \times 3 \times g \times \frac{12}{13} = \frac{24}{13}g$$

put $F = \frac{24}{13}g$ in (2)

$$3a = T - \frac{24}{13}g - 3g \times \frac{5}{13}$$

rearranging gives $T = 3a + \frac{24}{13}g + \frac{15}{13}g$

$$T = 3a + \frac{39}{13}g$$

$$T = 3a + 3g \quad (4)$$

Put $T = 3a + 3g$ in (1)

$$7a = 7g - (3a + 3g)$$

$$7a + 3a = 7g - 3g$$

$$10a = 4g$$

$$a = \frac{4g}{10} = 3.92\,ms^{-2}$$

b) $s = 1m$
$u = 0\,ms^{-1}$
$v = ?$
$a = 3.92\,ms^{-2}$
$t = ?$

$$v^2 = u^2 + 2as$$

$$v^2 = 0^2 + 2 \times 3.92 \times 1$$

$$v^2 = 7.84$$

$$v = 2.8\,ms^{-1}$$

c) (4) gives $T = 3a + 3g$

when string breaks, $T = 0$ so $0 = 3a + 3g$

$$a = -g$$

$s = ?$
$u = 2.8\,ms^{-1}$
$v = 0\,ms^{-1}$
$a = -g$
$t = ?$

$$v = u + at$$

$$0 = 2.8 - 9.8t$$

$$t = \frac{2.8}{9.8} = 0.286 \text{ secs (3sf)}$$

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7.

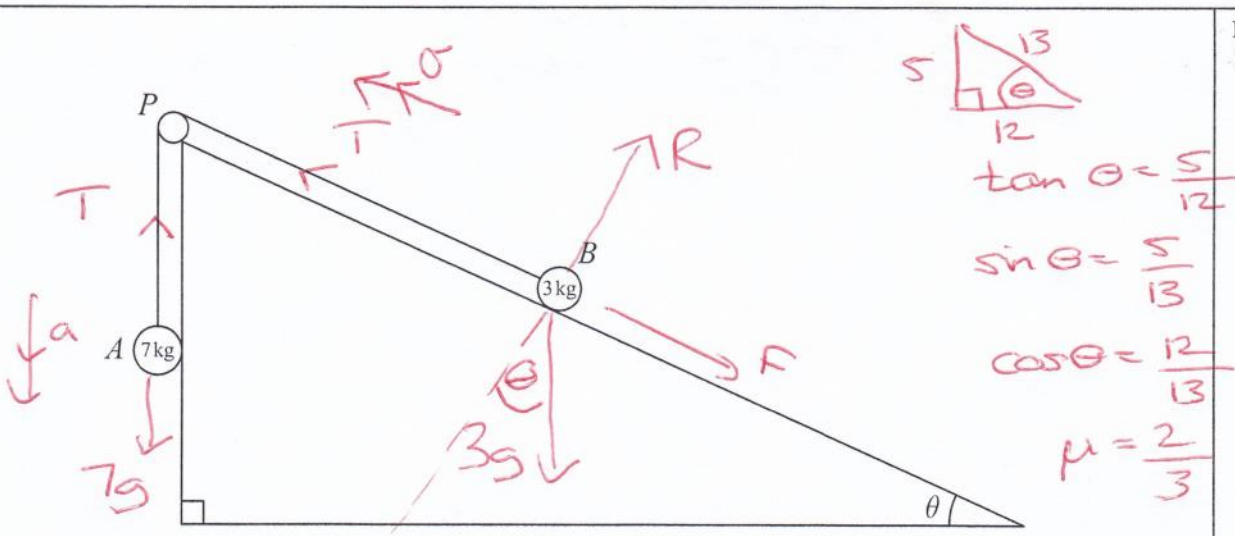


Figure 4

Two particles A and B, of mass 7 kg and 3 kg respectively, are attached to the ends of a light inextensible string. Initially B is held at rest on a rough fixed plane inclined at angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$. The part of the string from B to P is parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley, P, fixed at the top of the plane. The particle A hangs freely below P, as shown in Figure 4. The coefficient of friction between B and the plane is $\frac{2}{3}$. The particles are released from rest with the string taut and B moves up the plane.

(a) Find the magnitude of the acceleration of B immediately after release.

(10)

(b) Find the speed of B when it has moved 1 m up the plane.

(2)

When B has moved 1 m up the plane the string breaks. Given that in the subsequent motion B does not reach P,

(c) find the time between the instants when the string breaks and when B comes to instantaneous rest.

(4)

a) Equation of motion for A

$7a = 7g - T$ (1)

Equation of motion for B parallel to plane

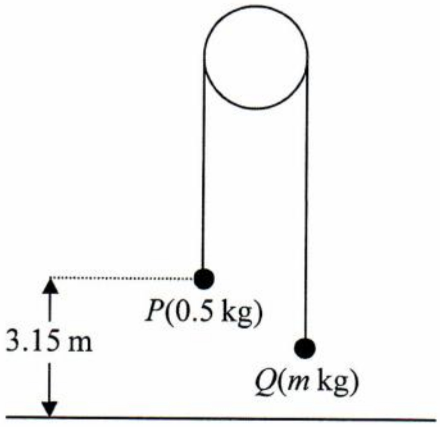
$3a = T - F - 3g\sin\theta$ (2)

Equation of motion perpendicular to plane

$R - 3g\cos\theta = 0$ (3)

Leave blank

6. **Figure 4**



Two particles P and Q have mass 0.5 kg and m kg respectively, where $m < 0.5$. The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initially P is 3.15 m above horizontal ground. The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in Figure 4. After P has been descending for 1.5 s, it strikes the ground. Particle P reaches the ground before Q has reached the pulley.

(a) Show that the acceleration of P as it descends is 2.8 m s^{-2}.

(3)

(b) Find the tension in the string as P descends.

(3)

(c) Show that $m = \frac{5}{18}$.

(4)

(d) State how you have used the information that the string is inextensible.

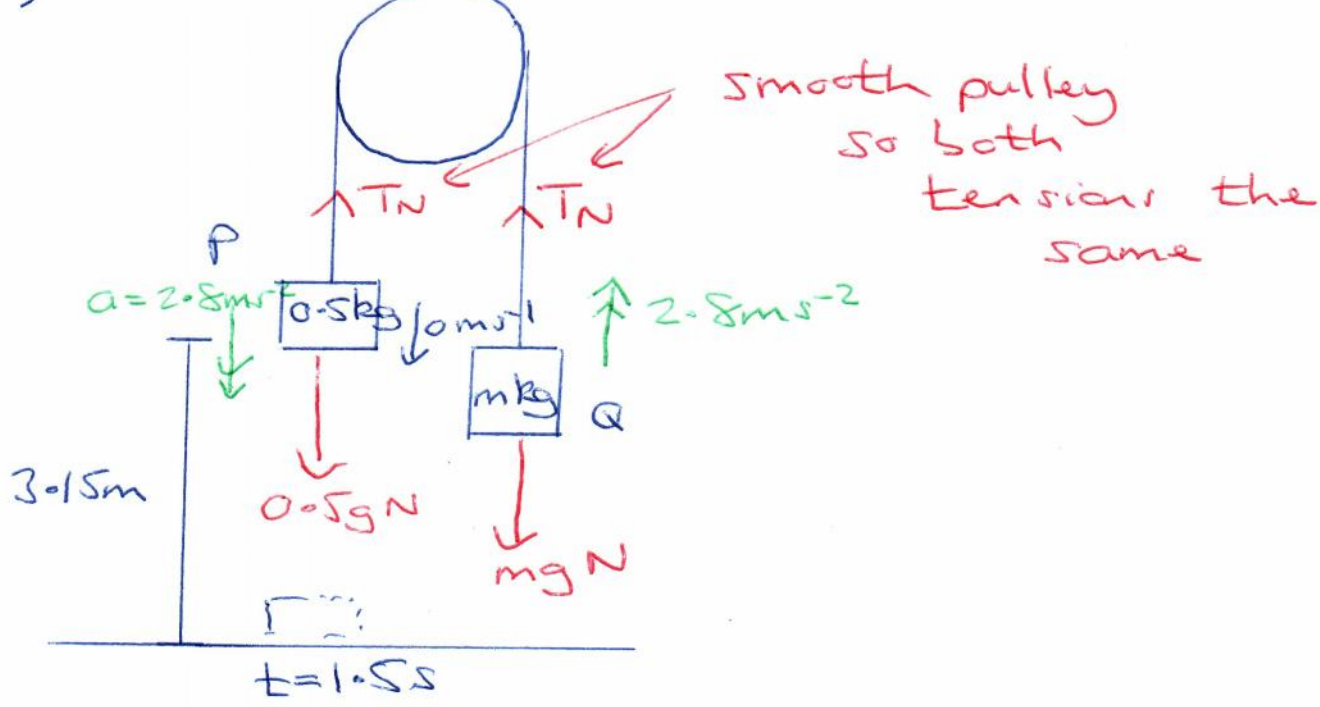
(1)

When P strikes the ground, P does not rebound and the string becomes slack. Particle Q then moves freely under gravity, without reaching the pulley, until the string becomes taut again.

(e) Find the time between the instant when P strikes the ground and the instant when the string becomes taut again.

(6)

6a)



a) s u v a t

$+\downarrow$ $s = 3.15m$, $u = 0 ms^{-1}$, $t = 1.5s$, $a = ?$

$$s = ut + \tfrac{1}{2}at^2$$

$\therefore$ $3.15 = 0 \times 1.5 + \tfrac{1}{2} \times a \times 1.5^2$

$\therefore$ $2(3.15) = (1.5)^2 a$

$\therefore$ $a = \dfrac{2(3.15)}{(1.5)^2}$

$a = 2.8 ms^{-2}$

b) $R(\downarrow)$ equation of motion for P

$$0.5g - T = 0.5 \times 2.8$$

Force = mass × acceleration

$\therefore$ $T = 0.5g - 1.4$

$\therefore$ $T = 3.5 N$

c) As string in inextensible, as P moves downwards, Q will immediately move upwards

consider Q

R (↑)

$T - mg = m \times 2.8$

$3.5 - mg = 2.8m$

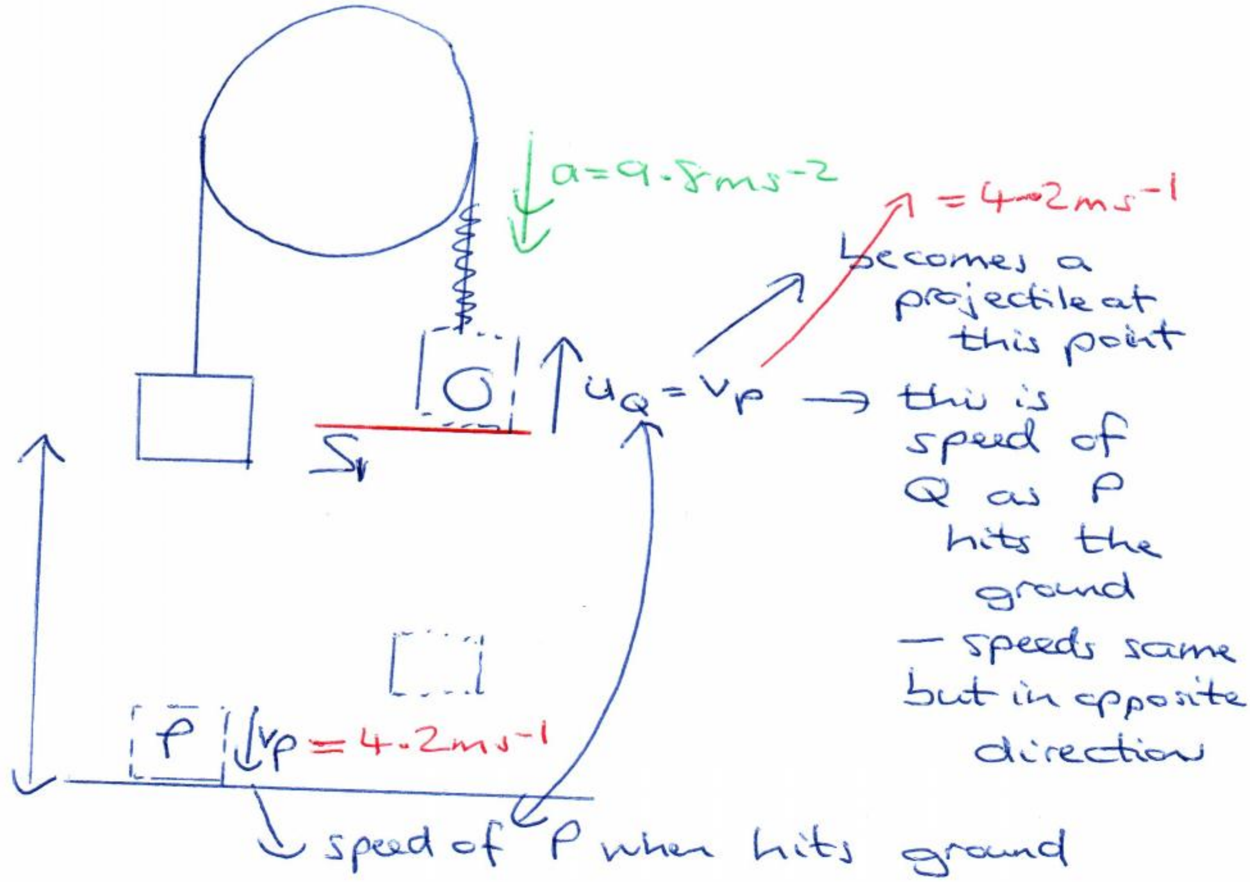
$\therefore 3.5 = 2.8m + 9.8m$

$\therefore 3.5 = 12.6m$

$\therefore m = \frac{3.5}{12.6} = \frac{35}{126} = \frac{5}{18}$

d) As string is inextensible, as P moves downwards, Q will move immediately upwards with the opposite acceleration of P. As P has acceleration 2.8 ms^{-2} down, Q will have acceleration 2.8 ms^{-2} upwards.

6e)



Firstly consider particle P $\downarrow +ve$

$v = v_p$, $u = 0\ ms^{-1}$

$a = 2.8 ms^{-2}$, $t = 1.5\ s$

$v = u + at$

$v_p = 0 + 2.8 \times 1.5$

$v_p = 4.2\ ms^{-1}$

Consider motion of Q $\uparrow +$

$s = 0$ (as it goes up from point S and back down, "displacement" will be ZERO to get back to starting point S_1)

$u = 4.2 ms^{-1}$

$a = -9.8 ms^{-2}$

$t = ?$

Using $s = ut + \frac{1}{2}at^2$

$\therefore 0 = 4.2t - 4.9t^2$ (x through by 10)

$\therefore 0 = 42t - 49t^2$

$\therefore 0 = 7t(6 - 7t)$

$\therefore 7t = 0$ or $6 - 7t = 0$

initial time

$t = \frac{6}{7}\ s$

Time when it becomes taut again is $t = \frac{6}{7}$ second

8.

Figure 4

Two particles P and Q, of mass 2 kg and 3 kg respectively, are joined by a light inextensible string. Initially the particles are at rest on a rough horizontal plane with the string taut. A constant force **F** of magnitude 30 N is applied to Q in the direction PQ, as shown in Figure 4. The force is applied for 3 s and during this time Q travels a distance of 6 m. The coefficient of friction between each particle and the plane is μ. Find

(a) the acceleration of Q,

(2)

(b) the value of μ,

(4)

(c) the tension in the string.

(4)

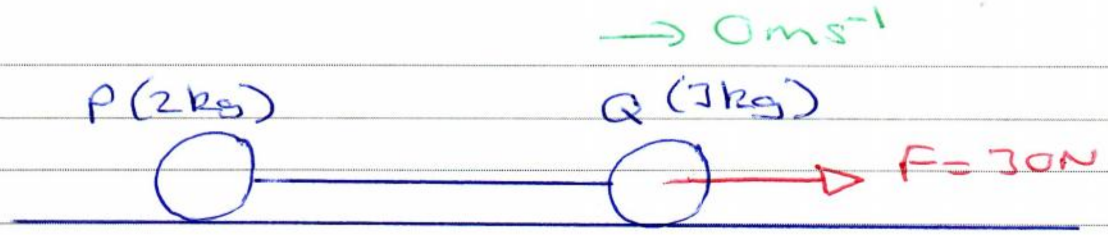
(d) State how in your calculation you have used the information that the string is inextensible.

(1)

When the particles have moved for 3 s, the force **F** is removed.

(e) Find the time between the instant that the force is removed and the instant that Q comes to rest.

(4)



$s = ut + \frac{1}{2}at^2$

a)

$s = 6m, u = 0 ms^{-1}, v = ?, a = ?, t = 3s$

$6 = 0 \times 3 + \frac{1}{2} \times a \times 3^2$

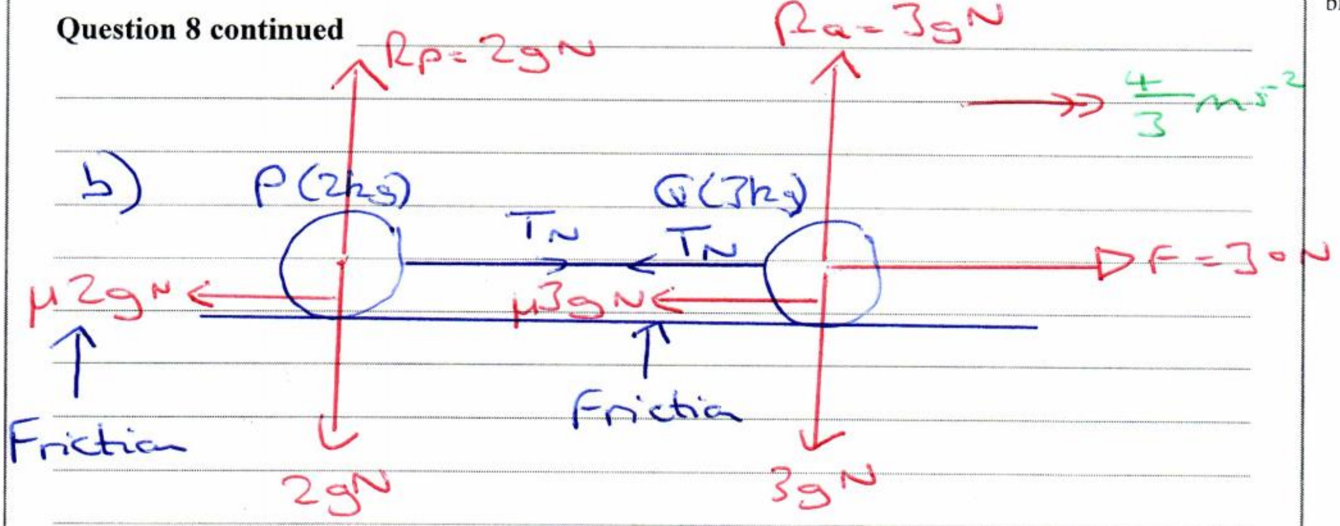
$6 = \frac{9}{2} a$

$\therefore a = \frac{2 \times 6}{9} = \frac{12}{9} = \frac{4}{3} ms^{-2}$

Leave blank

Question 8 continued

b)



Tension is string is T N

R ($\rightarrow$): Force = mass x acceleration

$30 - \mu 3g - T + T - \mu 2g = 5 \times \left(\frac{4}{3}\right)$

overall mass (3+2)

acceleration

(x by 3)

$90 - 9\mu g - 3T + 3T - 6\mu g = 20$

$\therefore 90 - 15\mu g = 20$

$\therefore 70 = 15\mu g$

$\therefore \mu = \frac{70}{15g} = \frac{10}{21} = 0.476$ (3sf)

Question 8 continued Using resultant force = mass x acceleration

c) consider particle P (but could do with Q to get same answer)

$R(\rightarrow)$:

$$T - \mu 2g = \underset{\text{mass}}{2}\left(\times \underset{\text{acceleration}}{\tfrac{4}{3}}\right)$$

$$T - 2g\left(\frac{10}{21}\right) = \frac{8}{3}$$

$$\therefore T = \frac{8}{3} + \frac{20g}{21}$$

$$\therefore T = 12\text{ N}$$

d) The particles move with the same acceleration

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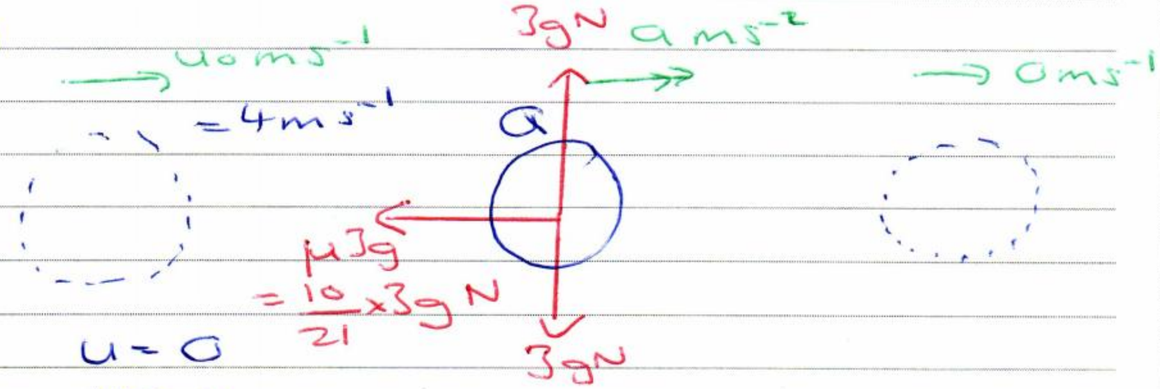
Question 8 continued

e)



$u = 0$

$v = u_0$

$t = 3$ s

$a = \frac{4}{3}$ ms^{-2}

$s = 6$m

using $v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times \frac{4}{3} \times 6$

$v^2 = 16$

$v = 4$ ms^{-1}

Now need to find the acceleration (or deceleration) of Q

R ($\rightarrow$):

$-\frac{10}{21} \times 3g = 3g \times a$

$\therefore a = -\frac{10g}{21}$ ms^{-2}

To get time, $v = u + at$

$\therefore 0 = 4 + \left(-\frac{10g}{21}\right) \times t$

$\frac{10g}{21} t = 4 \quad \therefore t = \frac{84}{10g} = \frac{6}{7}$ seconds

$= 0.856$ s (3sf) **(Total 15 marks)**

Q8

TOTAL FOR PAPER: 75 MARKS

END

8a) continued May 2010

put $a = 1.4$ in (2)

$T = (0.3 \times 1.4) + (0.3 \times 9.8)$

$T = 3.36\ N$

a) Tension in string after release is $3.36\ N$

b) acceleration of A after release $= 1.4\ ms^{-2}$

c) $s = ?$ $u = 0\ ms^{-1}$, $v = ?$, $a = 1.4\ ms^{-2}$, $t = 0.5$ secs

↑ +ve

Find speed at time $t = 0.5$

$v = u + at$

$v = 0 + (1.4 \times 0.5)$

$v = 0.7\ ms^{-1}$ when string breaks

Next find its height when string breaks

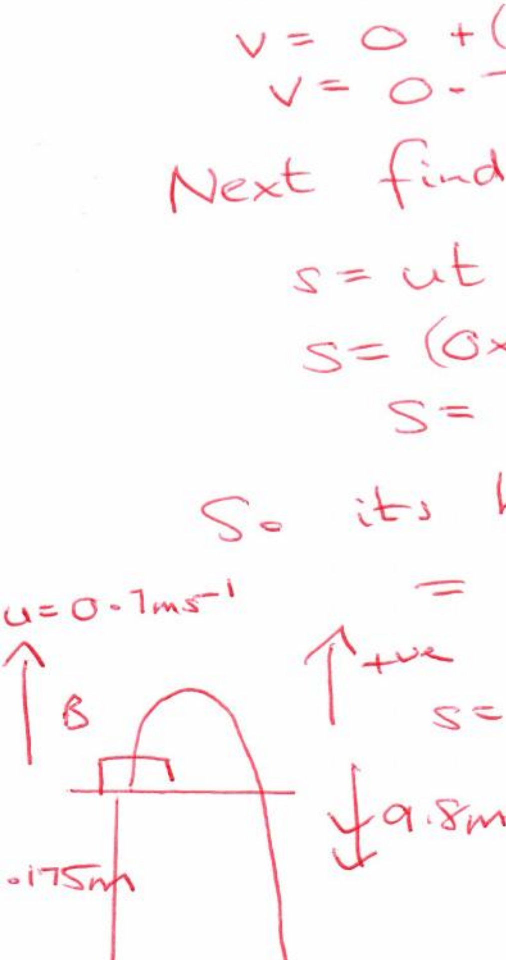
$s = ut + \frac{1}{2}at^2$

$s = (0 \times 0.5) + \frac{1}{2} \times 1.4 \times 0.5^2$

$s = 0.175\ m$

So its height when string breaks

$= 1 + 0.175 = 1.175\ m$



↑ +ve

$s = -1.175\ m$, $u = 0.7\ ms^{-1}$, $v = ?$,

↓ $9.8\ ms^{-2}$ $a = -9.8\ ms^{-2}$, $t = ?$

Find speed that it hits ground

$v^2 = u^2 + 2as$

$v^2 = (0.7)^2 + 2(-9.8)(-1.175)$

$v^2 = 23.52$

$v = \pm 4.8497423\ ms^{-1}$

May 2010

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8.

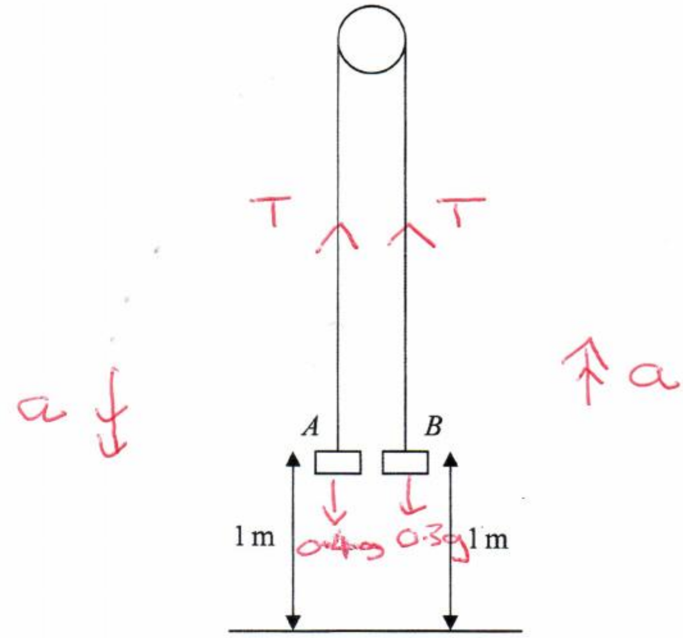

Figure 3

Two particles A and B have mass 0.4 kg and 0.3 kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed above a horizontal floor. Both particles are held, with the string taut, at a height of 1 m above the floor, as shown in Figure 3. The particles are released from rest and in the subsequent motion B does not reach the pulley.

(a) Find the tension in the string immediately after the particles are released. **(6)**

(b) Find the acceleration of A immediately after the particles are released. **(2)**

When the particles have been moving for 0.5 s, the string breaks.

(c) Find the further time that elapses until B hits the floor. **(9)**

a) Particle A R $(\downarrow)$

$-T + 0.4g = 0.4 \times a$. (1)

Particle B R $(\uparrow)$

$T - 0.3g = 0.3 \times a$ (2)

Solve (1) and (2) simultaneously

(1)+(2) gives $0.1g = 0.7a$

$a = \dfrac{0.1 \times 9.8}{0.7} = 1.4\,\text{ms}^{-2}$

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May 2010

↑ +ve

8c) continued

$v = u + at$

$t = \dfrac{v - u}{a}$

with
$s = -1.175m$, $u = 0.7 ms^{-1}$,
$v = -4.8497423 ms^{-1}$
$a = -9.8 ms^{-2}$
$t = ?$

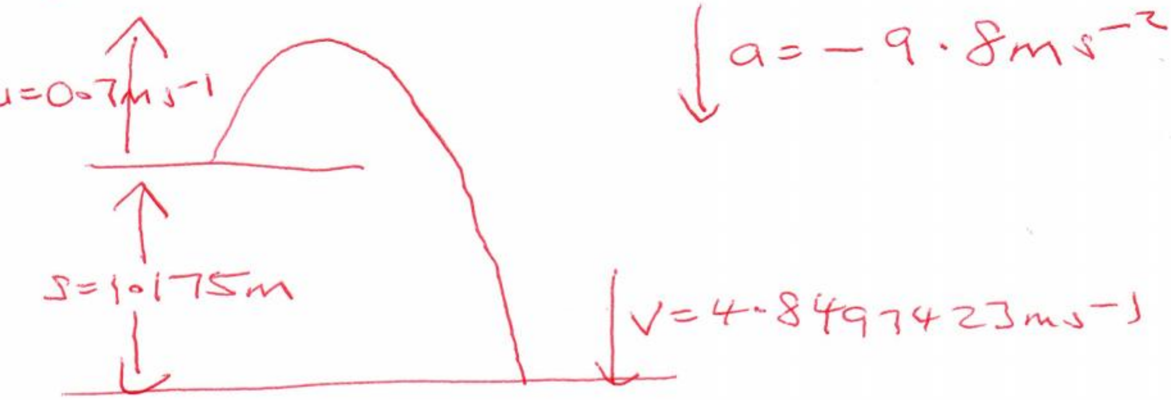
$$t = \frac{-4.8497423 - 0.7}{-9.8}$$

$t = 0.5663002$

$t = 0.566$ seconds (3 sf)

Further time until it hits the floor is 0.566 seconds (3 sf)

↑ +ve



May 2011

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6.

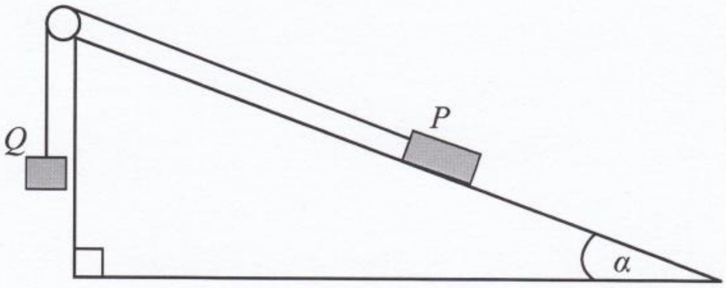


Figure 2

Two particles P and Q have masses 0.3 kg and m kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed rough plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{2}$.

The string lies in a vertical plane through a line of greatest slope of the inclined plane. The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut, as shown in Figure 2.

The system is released from rest and Q accelerates vertically downwards at $1.4\ \mathrm{m\ s^{-2}}$. Find

(a) the magnitude of the normal reaction of the inclined plane on P,

(2)

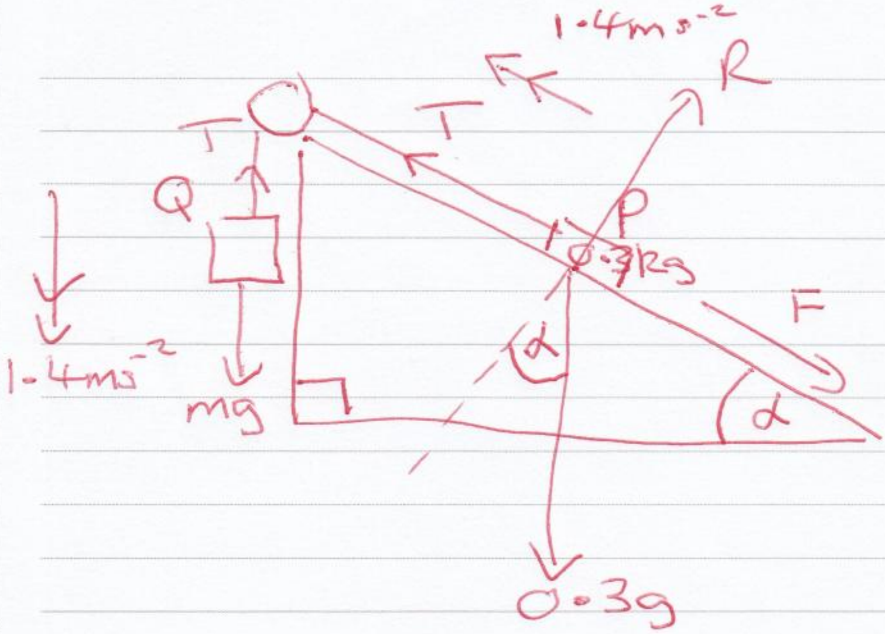
(b) the value of m.

(8)

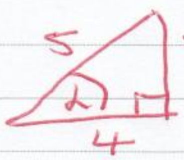
When the particles have been moving for 0.5 s, the string breaks. Assuming that P does not reach the pulley,

(c) find the further time that elapses until P comes to instantaneous rest.

(6)



$\tan\alpha = \frac{3}{4}$



$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

May 2011

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Question 6 continued

a) Equation of motion for Q

R(↓) $m \times 1.4 = mg - T$ (1)

Equation of motion for P

R (↖) parallel to plane

$0.3 \times 1.4 = T - F - 0.3g\sin\alpha$ (2)

(3) $F = \mu R$

$F = \frac{1}{2}R$

Equation of motion perpendicular to plane

$0 = R - 0.3g\cos\alpha$ (4)

$R = 0.3 \times 9.8 \times \frac{4}{5}$

$R = 2.352\,N$

Normal reaction of plane on P is $2.352\,N$

b) in (3)

$F = \frac{1}{2} \times R = \frac{1}{2} \times 2.352 = 1.176\,N$

in (2) gives

$0.42 = T - 1.176 - 0.3 \times 9.8 \times \frac{3}{5}$

$T = 0.42 + 1.176 + (0.3 \times 9.8 \times 0.6)$

$T = 3.36\,N$

May 2011

Question 6 continued

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in (1) gives

$1.4m = 9.8m - 3.36$

$3.36 = 9.8m - 1.4m$

$3.36 = 8.4m$

$m = \frac{3.36}{8.4} = 0.4\,kg$

c) When string breaks, $T = 0\,N$
Find speed when string breaks

$s = ?$
$u = 0\,ms^{-1}$
$v = ?$
$a = 1.4\,ms^{-2}$
$t = 0.5\,secs$

$v = u + at$
$v = 0 + 1.4 \times 0.5$
$v = 0.7\,ms^{-2}$

Equation of motion for P ($T = 0$ in string)
($\rightarrow$ to find new acceleration)

$0.3 \times a = -F - 0.3 \times 9.8 \times \frac{3}{5}$

$a = \frac{-1.176 - (0.3 \times 9.8 \times 0.6)}{0.3}$

$a = -9.8\,ms^{-2}$

comes to rest when $v = 0$

$s = ?$
$u = 0.7\,ms^{-1}$
$v = 0\,ms^{-1}$
$a = -9.8\,ms^{-2}$
$t = ?$

$v = u + at$

$t = \frac{v - u}{a}$

$t = \frac{0 - 0.7}{-9.8}$

$t = 0.0714$ seconds (3sf)

M1 JAN 2013

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7.

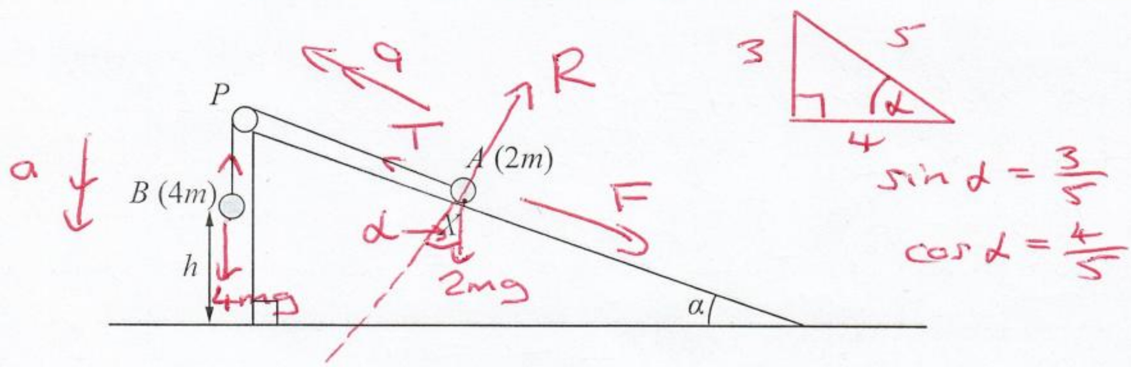

Figure 5

Figure 5 shows two particles A and B, of mass $2m$ and $4m$ respectively, connected by a light inextensible string. Initially A is held at rest on a rough inclined plane which is fixed to horizontal ground. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between A and the plane is $\frac{1}{4}$. The string passes over a small smooth pulley P which is fixed at the top of the plane. The part of the string from A to P is parallel to a line of greatest slope of the plane and B hangs vertically below P. The system is released from rest with the string taut, with A at the point X and with B at a height h above the ground.

For the motion until B hits the ground,

(a) give a reason why the magnitudes of the accelerations of the two particles are the same,

(1)

(b) write down an equation of motion for each particle,

(4)

(c) find the acceleration of each particle.

(5)

Particle B does not rebound when it hits the ground and A continues moving up the plane towards P. Given that A comes to rest at the point Y, without reaching P,

(d) find the distance XY in terms of h.

(6)

a) Particles connected by inextensible string

b) Equation of motion for B

R (↓)

$4ma = 4mg - T$ (1)

Equation of motion for A

R (↖) parallel to plane

$2ma = T - F - 2mg\sin\alpha$ (2)

7b) continued

(1) gives $T = 4mg - 4ma$

Need 3rd equation $F = \mu R$

$F = \frac{1}{4}R$ (3)

For A, motion perpendicular to plane R ($\nearrow$)

$2m \times 0 = R - 2mg\cos\alpha$ (4)

$R = 2mg\cos\alpha$

in (3) $F = \frac{1}{4} \times 2 \times mg\cos\alpha$

Put T and F in (2)

$2ma = T - F - 2mg\sin\alpha$

$2ma = (4mg - 4ma) - (\frac{1}{2}mg \times \frac{4}{5}) - 2mg \times \frac{3}{5}$

$2ma = 4mg - 4ma - \frac{2}{5}mg - \frac{6}{5}mg$

$4ma + 2ma = \frac{12}{5}mg$

$6\cancel{m}a = \frac{12}{5}\cancel{m}g$

c) $a = \frac{12 \times 9.8}{5 \times 6} = \underline{\underline{3.92\,ms^{-2}}}$

d) Find speed of B when it hits the ground R ($\downarrow$)

$s = h$

$u = 0\,ms^{-1}$

$v = ?$

$a = 3.92\,ms^{-2}$

$t = ?$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2 \times 3.92 \times h$

$v^2 = 7.84h$

$v = \sqrt{7.84h}$

M1 JAN 2013

7d) continued

When B hits ground, no more tension in string, so need to change equation of motion for A and find its new acceleration

R ($\nwarrow$) parallel to plane

$$2m \times a = -F - 2mg\sin\alpha$$

$$2ma = -\frac{2}{5}mg - 2mg \times \frac{3}{5}$$

$$2\cancel{m}a = -\frac{8}{5}\cancel{m}g$$

$$a = -\frac{4}{5}g = -\frac{4 \times 9.8}{5} = -7.84\ \text{ms}^{-2}$$

(decelerating)

R ($\nwarrow$)

$s = ?$
$u = \sqrt{7.84h}\ \text{ms}^{-1}$
$v = 0\ \text{ms}^{-1}$
$a = -7.84\ \text{ms}^{-2}$
$t = ?$

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a}$$

$$s = \frac{0^2 - 7.84h}{(2 \times -7.84)} = 0.5h$$

Distance XY = $h + 0.5h$

$= \underline{\underline{1.5h}}$ m

M1 MAY 2013

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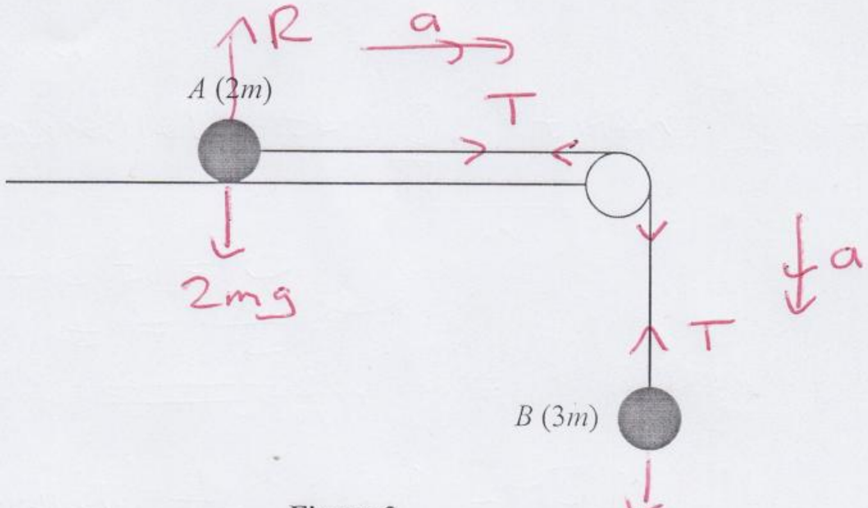


Figure 2

no friction

Two particles A and B have masses $2m$ and $3m$ respectively. The particles are attached to the ends of a light inextensible string. Particle A is held at rest on a smooth horizontal table. The string passes over a small smooth pulley which is fixed at the edge of the table. Particle B hangs at rest vertically below the pulley with the string taut, as shown in Figure 2. Particle A is released from rest. Assuming that A has not reached the pulley, find

(a) the acceleration of B,

(5)

(b) the tension in the string,

(1)

(c) the magnitude and direction of the force exerted on the pulley by the string.

(4)

a) Particle B $R(\downarrow)$

$3m \times a = 3mg - T$ (1)

Particle A $R(\rightarrow)$

$2m \times a = T$ (2)

sub (2) in (1)

$3ma = 3mg - 2ma$

$5\cancel{m}a = 3\cancel{m}g$

$a = \frac{3}{5}g = \frac{3}{5} \times 9.8 = 5.88 \text{ ms}^{-2}$

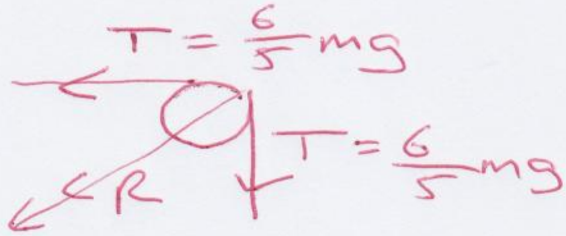
b) $T = 2 \times m \times \frac{3}{5}g = \frac{6}{5}mg$ N

$= 11.76m$ N

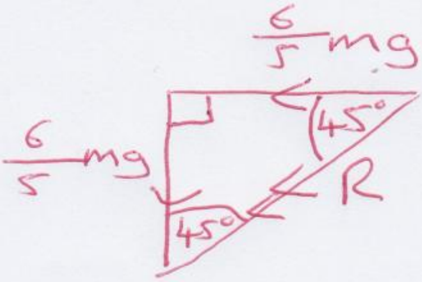
P 4 1 8 2 8 A 0 2 6 2 8

M1 MAY 2013

8c)



Resultant force $\rightarrow$ triangle of force



$$R^2 = \left(\frac{6}{5}mg\right)^2 + \left(\frac{6}{5}mg\right)^2$$

$$= \frac{36}{25}m^2g^2 + \frac{36}{25}m^2g^2$$

$$= \frac{72}{25}m^2g^2$$

$$R = \sqrt{\frac{72}{25}m^2g^2}$$

$$= \frac{\sqrt{72}}{5}mg = \frac{\sqrt{36}\sqrt{2}}{5}mg$$

$$= \underline{\underline{\frac{6\sqrt{2}}{5}mg \text{ N}}}$$

acting at 45° below horizontal