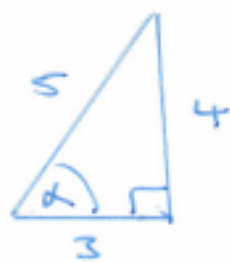


1.



$$\sin \alpha = \frac{4}{5}$$

$$\cos \alpha = \frac{3}{5}$$

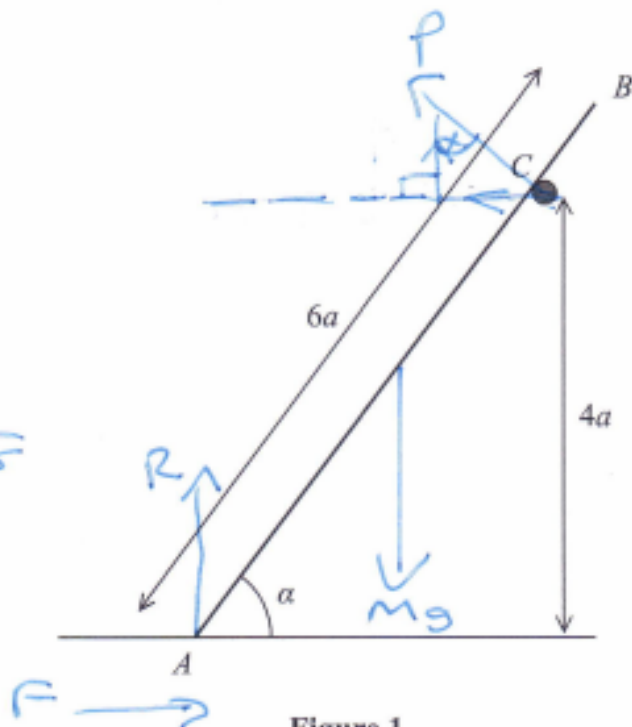


Figure 1



$$\sin \alpha = \frac{4a}{x}$$

$$x = \frac{4a}{\sin \alpha}$$

$$x = \frac{4a}{\frac{4}{5}}$$

$$x = 5a$$

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C .

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α , where $\sin \alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ .

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$ (3)

(b) Hence, or otherwise, find the value of μ . (7)

$$a) \quad R (\rightarrow) \quad F = P \sin \alpha \quad (1)$$

$$R (\uparrow) \quad R + P \cos \alpha - Mg = 0 \quad (2)$$

$$m(A) \quad \curvearrowright^{+ve} \quad 3a \cos \alpha \times Mg - 5a \times P = 0 \quad (3)$$

$$3\alpha \times \frac{3}{5} \times Mg = 5\alpha P$$

$$\frac{9}{25} Mg = P$$

(as required)

b) in (2)

$$R = Mg - P \cos \alpha$$

$$R = Mg - \frac{9}{25} Mg \times \frac{4}{5}$$

$$R = Mg - \frac{36}{125} Mg$$

$$R = \frac{98}{125} Mg$$

(1) gives $F = P \sin \alpha$

$$F = \frac{9}{25} Mg \times \frac{3}{5}$$

$$F = \frac{36}{125} Mg$$

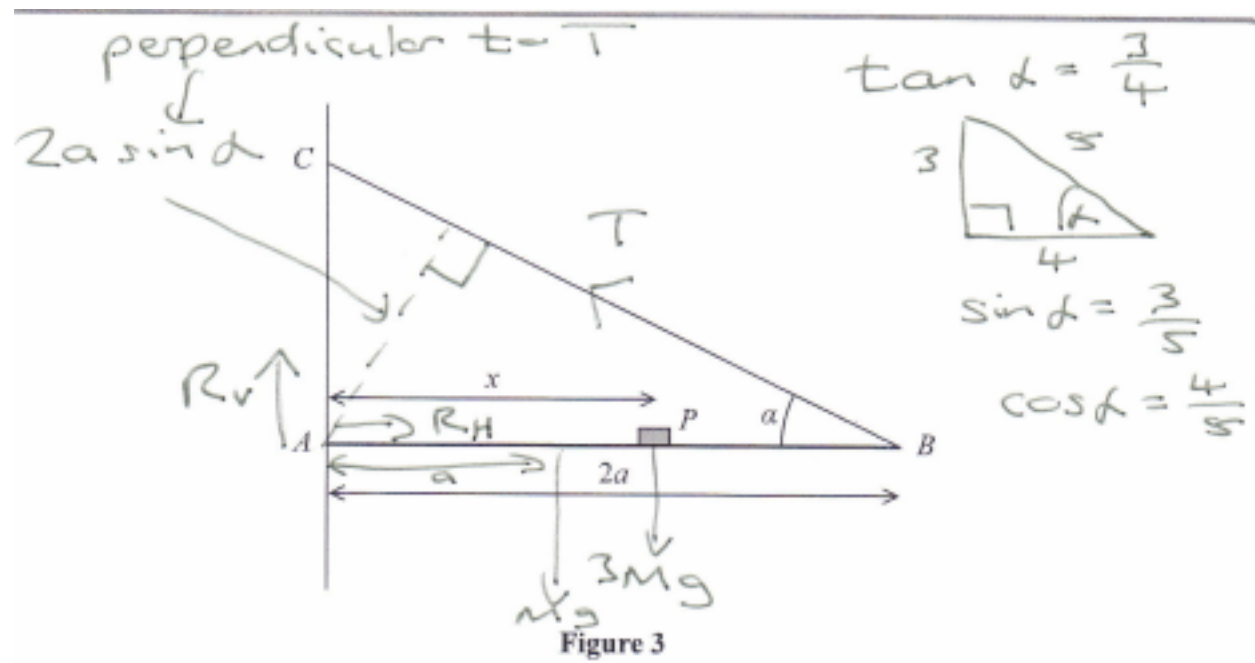
Limiting friction

$$F = \mu R$$

$$\mu = \frac{F}{R} = \frac{\frac{36}{125} Mg}{\frac{98}{125} Mg}$$

$$\mu = \frac{36}{98} = \frac{18}{49}$$

2.



A plank, AB , of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C , which is vertically above A .

A small block of mass $3M$ is placed on the plank at the point P , where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α , where $\tan \alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

(a) Using the model, show that the tension in the rope is $\frac{5Mg(3x + a)}{6a}$ (3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

(b) Find x in terms of a . (2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

(c) Find the value of $\tan \beta$ (5)

The rope will break if the tension in it exceeds $5Mg$.

(d) Explain how this will restrict the possible positions of P . You must justify your answer carefully. (3)

a) $m(A)$ \curvearrowright +ve

$$2a \sin \alpha \times T = a \times Mg + x \times 3Mg$$

$$2a \times \frac{3}{5} \times T = aMg + 3xMg$$

$$\frac{6}{5} a \times T = aMg + 3xMg$$

$$T = \frac{5}{6a} (aMg + 3xMg)$$

$$T = \frac{5Mg(a + 3x)}{6a}$$

(a is required)

b) R (\rightarrow)

$$R_H = T \cos \alpha$$

$$2Mg = \frac{5Mg(a + 3x)}{6a} \times \frac{4}{5}$$

$$\frac{2 \times 6a \times 5}{4 \times 5} = a + 3x$$

$$3a = a + 3x$$

$$2a = 3x$$

$$x = \frac{2}{3} a$$

c) $m(B)$ \curvearrowright +ve

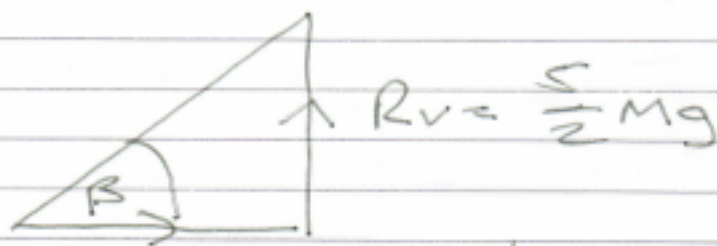
$$2a \times R_V = (2a - x) \times 3Mg + a \times Mg$$

$$2a R_V = (2a - \frac{2}{3}a) \times 3Mg + aMg$$

$$2a R_V = 4aMg + aMg$$

$$2a R_v = 5a Mg$$

$$R_v = \frac{5}{2} Mg$$



$$R_H = 2Mg$$

$$\tan \beta = \frac{\frac{5}{2}}{2} = \frac{5}{4}$$

c) If $T > 5 Mg$ rope will break

$$\frac{5 Mg (3x + a)}{6a} \leq 5 Mg$$

$$3x + a < 6a$$

$$3x < 5a$$

$$x \leq \frac{5}{3} a$$

For rope to not break

distance from A of the
block cannot be more
than $\frac{5a}{3}$ from A

6.

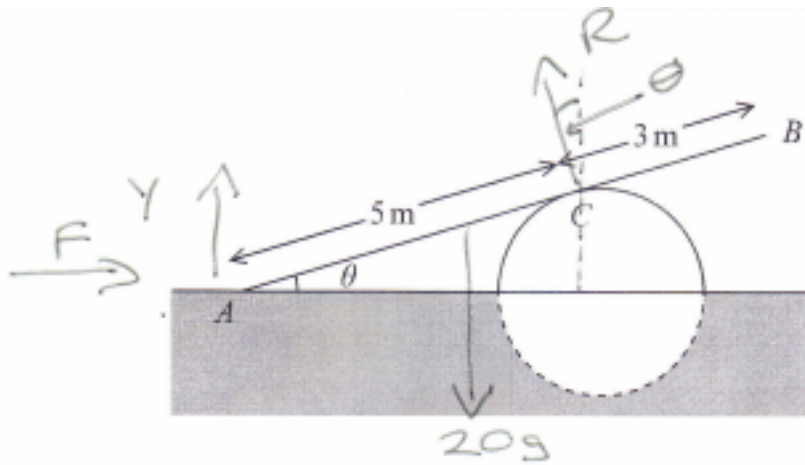


Figure 2

A ramp, AB , of length 8 m and mass 20 kg, rests in equilibrium with the end A on rough horizontal ground.

The ramp rests on a smooth solid cylindrical drum which is partly under the ground. The drum is fixed with its axis at the same horizontal level as A .

The point of contact between the ramp and the drum is C , where $AC = 5$ m, as shown in Figure 2.

The ramp is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle θ to the horizontal, where $\tan \theta = \frac{7}{24}$

The ramp is modelled as a uniform rod.

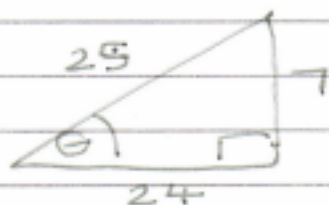
- (a) Explain why the reaction from the drum on the ramp at point C acts in a direction which is perpendicular to the ramp. (1)
- (b) Find the magnitude of the resultant force acting on the ramp at A . (9)

The ramp is still in equilibrium in the position shown in Figure 2 but the ramp is not now modelled as being uniform.

Given that the centre of mass of the ramp is assumed to be closer to A than to B ,

- (c) state how this would affect the magnitude of the normal reaction between the ramp and the drum at C . (1)

a) Ramp AB is a tangent to the cylindrical drum, only touching at C .



$$\sin \theta = \frac{7}{25}$$

$$\cos \theta = \frac{24}{25}$$

$$b) \quad m(A) \quad \overset{\text{+ve}}{\curvearrowright} \quad 4 \cos \theta \times 20g \quad (1)$$
$$- 5R = 0$$

$$4 \times \frac{24}{25} \times 20g = R$$

$$R = \frac{5}{5} \times 384g = 150.528N$$

$$R (\uparrow) \quad R \cos \theta + Y = 20g \quad (2)$$
$$Y = 20g - 150.528 \times \frac{24}{25}$$

$$Y = 51.49312N$$

$$R (\rightarrow) \quad F = R \sin \theta$$
$$F = 150.528 \times \frac{7}{25}$$
$$F = 42.14784N$$

Resultant force at A

$$= \sqrt{51.49312^2 + 42.14784^2}$$
$$= 66.54308$$
$$= 66.5N \quad (3 \text{ sf})$$

c) If centre of mass closer to A, using (1)

length 4cm would reduce

$4 \cos \theta$ would become smaller, so R would be smaller.