2. A curve C has equation

$$y = e^{2x}\tan x, \quad x \neq (2n+1)\frac{\pi}{2}.$$

(a) Show that the turning points on C occur where $\tan x = -1$.

(6)

(b) Find an equation of the tangent to C at the point where $x = 0$.

(2)

a) $y = e^{2x}\tan x$

Product rule $u = e^{2x}$ $\quad v = \tan x$

$\frac{du}{dx} = 2e^{2x}$ $\quad \frac{dv}{dx} = \sec^2 x$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= e^{2x}\sec^2 x + 2e^{2x}\tan x$

Turning point $\frac{dy}{dx} = 0$

$0 = \sec^2 x + 2\tan x$ $\quad [e^{2x} \neq 0]$

$0 = \tan^2 x + 1 + 2\tan x$ $\quad [\sec^2 x = \tan^2 x + 1]$

$0 = \tan^2 x + 2\tan x + 1$

$0 = (\tan x + 1)^2$

$\Rightarrow \tan x = -1.$

b) When $x = 0$ $y = 0$, $x = 0$ $\frac{dy}{dx} = 1$ $\Rightarrow y = x$

Leave blank

1. (a) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point where $x = 2$ on the curve with equation

$$y = x^2\sqrt{(5x-1)}.$$

(6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x.

(4)

a) $y = x^2(5x-1)^{1/2}$

Product Rule $\quad u = x^2 \qquad v = (5x-1)^{1/2}$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = \frac{1}{2}(5x-1)^{-1/2} \times 5$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

(no need to simplify) $\quad = \frac{5}{2}x^2(5x-1)^{-1/2} + (5x-1)^{1/2} \times 2x$

At $x = 2$ $\quad \frac{dy}{dx} = \frac{5}{2}(2^2)(10-1)^{-1/2} + (10-1)^{1/2} \times 4$

$\frac{dy}{dx} = \frac{46}{3}$

b) Quotient Rule.

$u = \sin 2x \qquad v = x^2$

$\frac{du}{dx} = 2\cos 2x \qquad \frac{dv}{dx} = 2x$

$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} = \frac{2x^2\cos 2x - 2x\sin 2x}{x^4}$

Leave blank

4. Find the equation of the tangent to the curve $x = \cos(2y + \pi)$ at $\left(0, \frac{\pi}{4}\right)$.

Give your answer in the form $y = ax + b$, where a and b are constants to be found.

(6)

$$\frac{dx}{dy} = -2\sin(2y + \pi)$$

$$\frac{dy}{dx} = -\frac{1}{2\sin(2y+\pi)}$$

At $\frac{\pi}{4}$

$$\frac{dy}{dx} = -\frac{1}{2\sin\left(\frac{3\pi}{2}\right)}$$

$$= \frac{1}{2}$$

Equation $(y - y_1) = m(x - x_1)$

At $(0, \pi/4)$ $\quad y - \frac{\pi}{4} = \frac{1}{2}x$

$$y = \frac{1}{2}x + \frac{\pi}{4}$$

$a = 1/2$

$b = \pi/4$

Leave blank

7.
$$f(x) = 3xe^x - 1$$

The curve with equation $y = f(x)$ has a turning point P.

(a) Find the exact coordinates of P.

(5)

The equation $f(x) = 0$ has a root between $x = 0.25$ and $x = 0.3$

(b) Use the iterative formula

$$x_{n+1} = \frac{1}{3}e^{-x_n}$$

with $x_0 = 0.25$ to find, to 4 decimal places, the values of x_1, x_2 and x_3.

(3)

(c) By choosing a suitable interval, show that a root of $f(x) = 0$ is $x = 0.2576$ correct to 4 decimal places.

(3)

a) Turning point when $f'(x) = 0$

Differentiate with product rule

$$\frac{dy}{dx} = 3e^x + 3xe^x$$

$$3e^x + 3xe^x = 0$$

$$3e^x(1+x) = 0 \qquad e^x \text{ cannot } = 0.$$

$$x = -1 \qquad \text{substitute into } f(x) \text{ for } y \text{ value.}$$

$$f(-1) = -3e^{-1} - 1 \qquad (-1, -3e^{-1} - 1)$$

b) $x_1 = 0.2596$ $x_2 = 0.2571$ $x_3 = 0.2578$

(use $\boxed{\text{ANS}}$ button to avoid rounding error)

c) Substitute in 0.25755 and 0.25765 to show a change of sign.

$f(0.25755) = -0.00038$ $f(0.25765) = 0.000109$ Change of sign

$\Rightarrow x = 0.2576$ is a root.

Leave blank

4. (i) Given that $y = \dfrac{\ln(x^2+1)}{x}$, find $\dfrac{dy}{dx}$.

(4)

(ii) Given that $x = \tan y$, show that $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$.

(5)

$y = \dfrac{\ln(x^2+1)}{x}$ Quotient Rule.

$u = \ln(x^2+1)$ $v = x$

$\dfrac{du}{dx} = \dfrac{2x}{x^2+1}$ $\dfrac{dv}{dx} = 1$

$\dfrac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$= \dfrac{x\left(\frac{2x}{x^2+1}\right) - \ln(x^2+1) \cdot 1}{x^2}$

$= \dfrac{2}{x^2+1} - \dfrac{1}{x^2}\ln(x^2+1)$

(ii) $x = \tan y$

$\dfrac{dx}{dy} = \sec^2 y$

$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y}$ $(\sec^2 y = 1 + \tan^2 y)$

$\dfrac{dy}{dx} = \dfrac{1}{1+\tan^2 y}$

$(x = \tan y)$

$\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$

Leave blank

7. (a) By writing $\sec x$ as $\dfrac{1}{\cos x}$, show that $\dfrac{\mathrm{d}(\sec x)}{\mathrm{d}x} = \sec x \tan x$.

(3)

Given that $y = \mathrm{e}^{2x} \sec 3x$,

(b) find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

(4)

The curve with equation $y = \mathrm{e}^{2x} \sec 3x$, $-\frac{\pi}{6} < x < \frac{\pi}{6}$, has a minimum turning point at (a, b).

(c) Find the values of the constants a and b, giving your answers to 3 significant figures.

(4)

a) $y = \sec x = \dfrac{1}{\cos x} = (\cos x)^{-1}$

$y = (\cos x)^{-1}$

$\dfrac{dy}{dx} = -(\cos x)^{-2}\ (-\sin x)$

$= \dfrac{\sin x}{\cos^2 x} \quad \left(= \dfrac{\sin x}{\cos x \cos x}\right)$

$\dfrac{dy}{dx} = \sec x \tan x$

(b) $y = e^{2x} \sec 3x$

Product Rule $\quad u = e^{2x} \quad v = \sec 3x$

$\dfrac{du}{dx} = 2e^{2x} \quad \dfrac{dv}{dx} = 3\sec 3x \tan 3x$

$\dfrac{dy}{dx} = 2e^{2x} \sec 3x + 3e^{2x} \sec 3x \tan x.$

Leave blank

Question 7 continued

c) Turning point $\frac{dy}{dx} = 0$

$$e^{2x}\sec 3x\,(2 + 3\tan 3x) = 0$$

$(e^{2x} \neq 0 \quad \sec 3x \neq 0)$

$$2 + 3\tan 3x = 0$$

$$\tan 3x = -\frac{2}{3}$$

$$3x = -0.588$$

$$x = -0.196$$

Substitute to find y

$$y = e^{2(-0.196)}\sec(3x^{-0.196})$$

$$= 0.812$$

$$x = -0.196 \quad y = 0.812$$

Turn over

7. The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}.$$

(*a*) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6\sin 2x + 4\cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(*b*) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

---

a) quotient rule.

$u = 3 + \sin 2x \qquad v = 2 + \cos 2x$

$\frac{du}{dx} = 2\cos 2x \qquad \frac{dv}{dx} = -2\sin 2x$

$$\frac{dy}{dx} = \frac{(2 + \cos 2x)(2\cos 2x) - (-2\sin 2x)(3 + \sin 2x)}{(2 + \cos 2x)^2}$$

$$= \frac{4\cos 2x + 2\cos^2 2x + 6\sin 2x + 2\sin^2 2x}{(2 + \cos 2x)^2}$$

$$= \frac{4\cos 2x + 6\sin 2x + 2(\cos^2 2x + \sin^2 2x)}{(2 + \cos 2x)^2}$$

$(\cos^2 2x + \sin^2 2x = 1)$

$$= \frac{4\cos 2x + 6\sin 2x + 2}{(2 + \cos 2x)^2}$$

(b) When $x = \pi/2$ $y = \dfrac{3 + \sin 2(\pi/2)}{2 + \cos 2(\pi/2)}$

Substitute π

$\downarrow$

$\underline{y = 3}$

$x = \pi/2$ $\dfrac{dy}{dx} = -2$

$\Rightarrow$ $y - 3 = -2\left(x - \pi/2\right)$

$y - 3 = -2x + \pi$

$\underline{y + 2x = \pi + 3}$

$y = -2x + \pi + 3$

$a = -2$

$b = \pi + 3$

8. Given that

$$\frac{d}{dx}(\cos x) = -\sin x,$$

(*a*) show that $\dfrac{d}{dx}(\sec x) = \sec x \tan x$.

(3)

Given that

$$x = \sec 2y,$$

(*b*) find $\dfrac{dx}{dy}$ in terms of y.

(2)

(*c*) Hence find $\dfrac{dy}{dx}$ in terms of x.

(4)

TOTAL FOR PAPER: 75 MARKS

END

8)

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$= \frac{d}{dx}(\cos x)^{-1}$$

$$= -(\cos x)^{-2} \cdot -\sin x$$

$$= \frac{\sin x}{(\cos x)^2}$$

$$= \frac{\sin x}{(\cos x)(\cos x)} = \underline{\tan x \sec x}$$

(b)

$$x = \sec 2y$$

$$\frac{dx}{dy} = 2\tan 2y \sec 2y$$

(c) $\frac{dy}{dx} = \frac{1}{2\tan 2y \sec 2y}$

but

$x = \sec 2y$ $\qquad \frac{dy}{dx} = \frac{1}{2x \tan 2y}$

but

$1 + \tan^2 y = \sec^2 y$

so $1 + \tan^2 2y = \sec^2 2y$

$\tan^2 2y = \sec^2 2y - 1$

$x = \sec 2y$ $\qquad \tan^2 2y = x^2 - 1$

$\tan 2y = \sqrt{x^2 - 1}$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2x\sqrt{x^2-1}}$$

C3 Jan 2012

Leave blank

1. Differentiate with respect to x, giving your answer in its simplest form,

(a) $x^2 \ln(3x)$

(4)

(b) $\dfrac{\sin 4x}{x^3}$

(5)

1a) Product Rule

$$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$u = x^2 \qquad v = \ln(3x)$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = \frac{1}{x}$

$$\frac{dy}{dx} = 2x\ln(3x) + x^2 \cdot \frac{1}{x}$$

$$= 2x\ln(3x) + x$$

(b) Quotient Rule.

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = \sin 4x \qquad v = x^3$

$\frac{du}{dx} = 4\cos 4x \qquad \frac{dv}{dx} = 3x^2$

$$\frac{dy}{dx} = \frac{4x^3\cos 4x - 3x^2\sin 4x}{x^6}$$

$$= \frac{4x\cos 4x - 3\sin 4x}{x^4}$$

P 4 0 0 8 4 A 0 2 2 4

C3 Jan 2012

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4. The point P is the point on the curve $x = 2\tan\left(y + \frac{\pi}{12}\right)$ with y-coordinate $\frac{\pi}{4}$.

Find an equation of the normal to the curve at P.

(7)

4) $\frac{dx}{dy} = 2\sec^2\left(y + \frac{\pi}{12}\right)$

when $y = \pi/4$ $\quad \frac{dx}{dy} = \frac{2}{\cos^2\left(\frac{\pi}{4} + \frac{\pi}{12}\right)}$

$= 8$

$\frac{dy}{dx}_{\text{TANGENT}} = \frac{1}{8}$

$\frac{dy}{dx}_{\text{normal}} = -8$

$x = 2\tan\left(\frac{\pi}{4} + \frac{\pi}{12}\right)$

$= 2\sqrt{3}$

$y = \pi/4$ (given)

$y - \frac{\pi}{4} = -8(x - 2\sqrt{3})$

P 4 0 0 8 4 A 0 8 2 4

Leave blank

3. A curve C has equation

$$y = x^2 e^x.$$

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation. **(3)**

(b) Hence find the coordinates of the turning points of C. **(3)**

(c) Find $\frac{d^2y}{dx^2}$. **(2)**

(d) Determine the nature of each turning point of the curve C. **(2)**

a) $u = x^2 \quad v = e^x$

$\frac{du}{dx} = 2x \quad \frac{dv}{dx} = e^x \quad \frac{dy}{dx} = x^2e^x + 2xe^x$

b) $\frac{dy}{dx} = 0 \quad 0 = (x^2 + 2x)\, e^x$

$x^2 + 2x = 0 \qquad e^x \neq 0$

$x = 0 \quad x = -2$ subs. x values into $y = x^2e^x$

$y = 0 \quad \cancel{x}y = 4e^{-2}$

$(0, 0) \quad (-2, 4e^{-2})$

c) $\frac{d^2y}{dx^2} = x^2e^x + 2xe^x + 2xe^x + 2e^x$

$= e^x(x^2 + 4x + 2)$

product rule

d) $x = 0 \quad \frac{d^2y}{dx^2} > 0$ minimum

$x = -2 \quad \frac{d^2y}{dx^2} < 0$ maximum

N 2 6 1 0 9 A 0 6 2 4

6. (*a*) Differentiate with respect to x,

(i) $e^{3x}(\sin x + 2\cos x)$,

(3)

(ii) $x^3 \ln(5x + 2)$.

(3)

Given that $y = \dfrac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(*b*) show that $\dfrac{dy}{dx} = \dfrac{20}{(x+1)^3}$.

(5)

(*c*) Hence find $\dfrac{d^2y}{dx^2}$ and the real values of x for which $\dfrac{d^2y}{dx^2} = -\dfrac{15}{4}$.

(3)

6a) Product Rule $\quad \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

$u = e^{3x} \qquad v = \sin x + 2\cos x$

$\dfrac{du}{dx} = 3e^{3x} \qquad \dfrac{dv}{dx} = \cos x - 2\sin x$

$$\frac{dy}{dx} = e^{3x}(\cos x - 2\sin x) + (\sin x + 2\cos x)3e^{3x}$$

(ii) Product Rule

$u = x^3 \qquad v = \ln(5x+2)$

$\dfrac{du}{dx} = 3x^2 \qquad \dfrac{dv}{dx} = \dfrac{5}{5x+2}$

$$\frac{dy}{dx} = \frac{5x^3}{5x+2} + 3x^2(\ln(5x+2))$$

b) Quotient Rule

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = 3x^2 + 6x - 7$ $\quad\quad$ $v = (x+1)^2$

$\frac{du}{dx} = 6x + 6$ $\quad\quad$ $\frac{dv}{dx} = 2(x+1)$

$$\frac{dy}{dx} = \frac{(6x+6)(x+1)^2 - 2(x+1)(3x^2+6x-7)}{(x+1)^4}$$

cancel $(x+1)$ everywhere

$$= \frac{(6x+6)(x+1) - 2(3x^2+6x-7)}{(x+1)^3}$$

$$= \frac{6x^2 + 6x + 6x + 6 - 6x^2 - 12x + 14}{(x+1)^3}$$

$$= \frac{2c}{(x+1)^3}$$

(c) $\frac{dy}{dx} = 2c(x+1)^{-3}$

$$\frac{d^2y}{dx^2} = -6c(x+1)^{-4}$$

$$= \frac{-6c}{(x+1)^4}$$

so $\frac{d^2y}{dx^2} = -\frac{15}{4} = \frac{-6c}{(x+1)^4}$

$-15(x+1)^4 = -24c$

$(x+1)^4 = 16$

$x+1 = \pm 2$

$x = 1$ and $x = -3$

Leave blank

3. Rabbits were introduced onto an island. The number of rabbits, P, t years after they were introduced is modelled by the equation

$$P = 80e^{\frac{1}{5}t}, \qquad t \in \mathbb{R}, t \geqslant 0$$

(a) Write down the number of rabbits that were introduced to the island. **(1)**

(b) Find the number of years it would take for the number of rabbits to first exceed 1000. **(2)**

(c) Find $\frac{dP}{dt}$. **(2)**

(d) Find P when $\frac{dP}{dt} = 50$. **(3)**

a) $t = 0 \qquad P = 80e^0$

$P = 80$

(b) $P = 1000 \Rightarrow 1000 = 80e^{1/5 t}$

$\frac{1000}{80} = e^{1/5 t}$

$\ln\left(\frac{1000}{80}\right) = 1/5\, t$

$t = 5\ln\left(\frac{1000}{80}\right)$

$t = 12.6$ or 13 years.

(c) $\frac{dP}{dt} = 16e^{1/5 t}$

(d) $50 = 16e^{1/5 t}$

$\frac{50}{16} = e^{1/5 t} \Rightarrow \ln\left(\frac{50}{16}\right) = 1/5\, t \Rightarrow t = 5\ln\left(\frac{50}{16}\right)$

$P = 80e^{1/5\left(5\ln\left(\frac{50}{16}\right)\right)}$ * do not round.

$P = 250$

Leave blank

4. (i) Differentiate with respect to x

(a) $x^2 \cos 3x$

(3)

(b) $\dfrac{\ln(x^2+1)}{x^2+1}$

(4)

(ii) A curve C has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\tfrac{1}{4}, \; y > 0$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

(i) (a) $u = x^2 \qquad v = \cos 3x$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dx} = -3\sin 3x$ product rule.

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$\frac{dy}{dx} = -3x^2\sin 3x + 2x\cos 3x$

b) $u = \ln(x^2+1) \qquad v = x^2+1$ quotient rule

$\frac{du}{dx} = \frac{2x}{x^2+1} \qquad \frac{dv}{dx} = 2x$

$\frac{dy}{dx} = \dfrac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

$\frac{dy}{dx} = \dfrac{\cancel{(x^2+1)}\frac{2x}{\cancel{x^2+1}} - \ln(x^2+1).2x}{(x^2+1)^2} = \dfrac{2x - 2x\ln(x^2+1)}{(x^2+1)^2}$

H34264A01028

Question 4 continued

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$$y = (4x+1)^{1/2}$$

Gradient (m) $\frac{dy}{dx} = \frac{1}{2}(4x+1)^{-1/2}.4$

$$= 2(4x+1)^{-1/2}$$

When $x = 2$ $\quad y = (4 \times 2 + 1)^{1/2}$

$$y = 3$$

When $x = 2$ $\quad \frac{dy}{dx} = 2(4 \times 2 + 1)^{-1/2}$

$$(m)\ \frac{dy}{dx} = 2/3$$

Using '$y = mx + c$'

$$3 = \frac{2}{3}.2 + c$$

$$c = \frac{5}{3}$$

$\therefore \quad y = \frac{2}{3}x + \frac{5}{3}$ $\quad$ ✗ 'Put in form $ax + by + c = 0$'

$$2x - 3y + 5 = 0$$

H34264A01128

2. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \qquad x \neq \frac{5}{3}.$$

The point P on C has x-coordinate 2.

Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

$$y = 3(5-3x)^{-2}$$

$$\frac{dy}{dx} = -6(5-3x)^{-3} \cdot -3$$

$$= 18(5-3x)^{-3}$$

$x = 2$ $\quad \frac{dy}{dx} = -18$

(TANGENT

Gradient Normal $= \frac{1}{18}$

when $x = 2$ $\quad y = \frac{3}{(5-3\times2)^2}$

$$y = 3$$

$$y - 3 = \frac{1}{18}(x-2)$$

$$18y - 54 = x - 2$$

$$x - 18y + 52 = 0$$

Leave blank

1. Differentiate with respect to x

(a) $\ln\left(x^2+3x+5\right)$ **(2)**

(b) $\dfrac{\cos x}{x^2}$ **(3)**

a) $\dfrac{2x+3}{x^2+3x+5}$

b) $u = \cos x \qquad v = x^2$

$\dfrac{du}{dx} = -\sin x \qquad \dfrac{dv}{dx} = 2x$

Quotient rule

$$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{x^2(-\sin x) - 2x\cos x}{x^4}$$

OR $= \dfrac{-x^2\sin x - 2x\cos x}{x^4}$

OR $= \dfrac{-x\sin x - 2\cos x}{x^3}$

P38159A0224

C3 June 2012

7. (a) Differentiate with respect to x,

(i) $x^{\frac{1}{2}}\ln(3x)$

(ii) $\dfrac{1-10x}{(2x-1)^5}$, giving your answer in its simplest form.

(6)

(b) Given that $x = 3\tan 2y$ find $\dfrac{dy}{dx}$ in terms of x.

(5)

7a) (i) $u = x^{1/2}$ $\quad v = \ln 3x$

$\dfrac{du}{dx} = \dfrac{1}{2}x^{-1/2}$ $\quad \dfrac{dv}{dx} = \dfrac{3}{3x}$

$= \dfrac{1}{x}$

$\dfrac{dy}{dx} = \dfrac{1}{2}x^{-1/2}\ln(3x) + x^{1/2}\left(\dfrac{1}{x}\right)$

$= \dfrac{\ln(3x)}{2x^{1/2}} + \dfrac{1}{x^{1/2}}$

(ii) Quotient Rule

$u = 1-10x$ $\quad v = (2x-1)^5$

$\dfrac{du}{dx} = -10$ $\quad \dfrac{dv}{dx} = 10(2x-1)^4$

$\dfrac{dy}{dx} = \dfrac{-10(2x-1)^5 - 10(2x-1)^4(1-10x)}{((2x-1)^5)^2}$

$= \dfrac{-10(2x-1)^{\cancel{5}} - 10(1-10x)\cancel{(2x-1)^4}}{(2x-1)^{\cancel{10}\,6}}$

Leave blank

Question 7 continued

$$\frac{dy}{dx} = \frac{-10(2x-1) - 10(1-10x)}{(2x-1)^6}$$

$$= \frac{-20x + 10 - 10 + 100x}{(2x-1)^6}$$

$$= \frac{80x}{(2x-1)^6}$$

(b)

$x = 3\tan 2y$

$$\frac{dx}{dy} = 6\sec^2 2y$$

$$\boxed{\frac{dy}{dx} = \frac{1}{6\sec^2 2y}}$$

$$\frac{dy}{dx} = \frac{1}{6}(\cos^2 2y)$$

Remember $\quad x = 3\tan 2y$

(square) $\quad x^2 = 9\tan^2 2y$

$$\frac{x^2}{9} = \tan^2 2y \qquad (\text{but } \tan^2 y + 1 = \sec^2 y)$$

so $\quad \dfrac{x^2}{9} = \sec^2 2y - 1$

substitute into $\dfrac{dy}{dx}$

so $\quad \sec^2 2y = \dfrac{x^2}{9} + 1$

P40686RA02532

Turn over

C3 June 2012

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Question 7 continued

$$\frac{dy}{dx} = \frac{1}{6\sec^2 2y}$$

$$\frac{dy}{dx} = \frac{1}{6\left(\frac{x^2}{9}+1\right)}$$

$$= \frac{1}{6\left(\frac{x^2+9}{9}\right)}$$

$$= \frac{9}{6(x^2+9)}$$

$$\frac{dy}{dx} = \frac{3}{2(x^2+9)}$$

P 4 0 6 8 6 R A 0 2 6 3 2

C3 JAN 2013

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1. The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(a) the value of w,

(2)

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

a) when $x = w$, $y = -32$

$-32 = (2x-3)^5$

5th root of -32 is -2

$-2 = 2x - 3$

$3 - 2 = 2x$

$1 = 2x$

$x = \frac{1}{2}$

So $w = \frac{1}{2}$

b) $y = (2x-3)^5$

$\frac{dy}{dx} = 2 \times 5(2x-3)^4 = 10(2x-3)^4$

at $x = \frac{1}{2}$, $\frac{dy}{dx} = 10\left(2 \times \frac{1}{2} - 3\right)^4$

$\frac{dy}{dx} = 10(1-3)^4 = 10(-2)^4 = 160$

$y - y_1 = m(x - x_1)$

$y - (-32) = 160\left(x - \frac{1}{2}\right)$

$y + 32 = 160x - 80$

$y = 160x - 80 - 32$

$\underline{\underline{y = 160x - 112}}$

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5. (i) Differentiate with respect to x

(a) $y = x^3 \ln 2x$

(b) $y = (x + \sin 2x)^3$

(6)

Given that $x = \cot y$,

(ii) show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-1}{1+x^2}$

(5)

(i) a) $y = x^3 \ln 2x$ $\quad u = x^3 \quad v = \ln 2x$

$\frac{du}{dx} = 3x^2 \quad \frac{dv}{dx} = \frac{1}{x}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} = x^3 \times \frac{1}{x} + 3x^2 \ln 2x$

$\frac{dy}{dx} = x^2 + 3x^2 \ln 2x$

b) $y = (x + \sin 2x)^3$

$\frac{dy}{dx} = 3(x + \sin 2x)^2 \times (1 + 2\cos 2x)$

(ii) $x = \cot y = \frac{\cos y}{\sin y}$

$u = \cos y \quad v = \sin y$

$\frac{du}{dy} = -\sin y \quad \frac{dv}{dy} = \cos y$

$\frac{dx}{dy} = \frac{v\frac{du}{dy} - u\frac{dv}{dy}}{v^2}$

$\frac{dx}{dy} = \frac{\sin y \times -\sin y - \cos y \times \cos y}{\sin^2 y}$

$\frac{dx}{dy} = \frac{-\sin^2 y - \cos^2 y}{\sin^2 y}$

5 (ii) (continued)

$$\frac{dx}{dy} = -1 - \frac{\cos^2 y}{\sin^2 y}$$

$$\frac{dx}{dy} = -1 - \cot^2 y \qquad \Big\{ x = \cot y$$

$$\frac{dx}{dy} = -1 - x^2$$

$$\frac{dx}{dy} = -(1+x^2)$$

$$\frac{dy}{dx} = \frac{1}{-(1+x^2)}$$

$$\frac{dy}{dx} = \frac{-1}{1+x^2}$$ as required

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5. Given that

$$x = \sec^2 3y, \qquad 0 < y < \frac{\pi}{6}$$

(a) find $\frac{dx}{dy}$ in terms of y.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{1}{6x(x-1)^{\frac{1}{2}}}$$

(4)

(c) Find an expression for $\frac{d^2y}{dx^2}$ in terms of x. Give your answer in its simplest form.

(4)

a) $x = \sec^2 3y = (\sec 3y)^2$

$\frac{dx}{dy} = 2(\sec 3y)^1 \times 3\sec 3y \tan 3y$

$= \underline{6\sec^2 3y \tan 3y}$

b) $\frac{dy}{dx} = \frac{1}{6\sec^2 3y \tan 3y}$

$\frac{dy}{dx} = \frac{1}{6x\sqrt{x-1}}$

$x = \sec^2 3y$

$\sec^2 3y = 1 + \tan^2 3y$

$\therefore x = 1 + \tan^2 3y$

$x - 1 = \tan^2 3y$

$\sqrt{x-1} = \tan 3y$

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5c) $\dfrac{dy}{dx} = \dfrac{1}{6x\sqrt{x-1}} = \dfrac{x^{-1}}{6(x-1)^{\frac{1}{2}}} \begin{matrix} \leftarrow u \\ \leftarrow v \end{matrix}$

$$\frac{d^2y}{dx^2} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = x^{-1}$ $\qquad v = 6(x-1)^{\frac{1}{2}}$

$u' = -1x^{-2}$ $\qquad v' = 3(x-1)^{-\frac{1}{2}}$

$$\frac{d^2y}{dx^2} = \frac{-6(x-1)^{\frac{1}{2}} \times x^{-2} - x^{-1} \cdot 3(x-1)^{-\frac{1}{2}}}{6^2 \times ((x-1)^{\frac{1}{2}})^2}$$

$$= \frac{\dfrac{-6(x-1)^{\frac{1}{2}}}{x^2} - \dfrac{3(x-1)^{-\frac{1}{2}}}{x}}{36(x-1)}$$

$$= \frac{\dfrac{-6(x-1)^{\frac{1}{2}}}{x^2} - \dfrac{3}{x(x-1)^{\frac{1}{2}}}}{36(x-1)}$$

$$= \frac{\dfrac{-6((x-1)^{\frac{1}{2}})^2 - 3x}{x^2(x-1)^{\frac{1}{2}}}}{36(x-1)} = \frac{-6(x-1) - 3x}{36x^2(x-1)^{\frac{3}{2}}}$$

$$= \frac{-6x + 6 - 3x}{36x^2(x-1)^{\frac{3}{2}}} = \frac{-9x + 6}{36x^2(x-1)^{\frac{3}{2}}}$$

$$= \frac{-\overset{1}{\cancel{3}}(3x-2)}{12\cancel{36}x^2(x-1)^{\frac{3}{2}}} = \underline{\underline{\frac{-3x+2}{12x^2(x-1)^{\frac{3}{2}}}}}$$