

Please check the examination details below before entering your candidate information

Candidate surname

Other names

**Pearson Edexcel
Level 3 GCE**

Centre Number

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Candidate Number

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Monday 19 October 2020

Afternoon

Paper Reference **9MA0/32**

Mathematics

Advanced

Paper 32: Mechanics

You must have:

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

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Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 5 questions.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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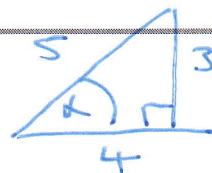
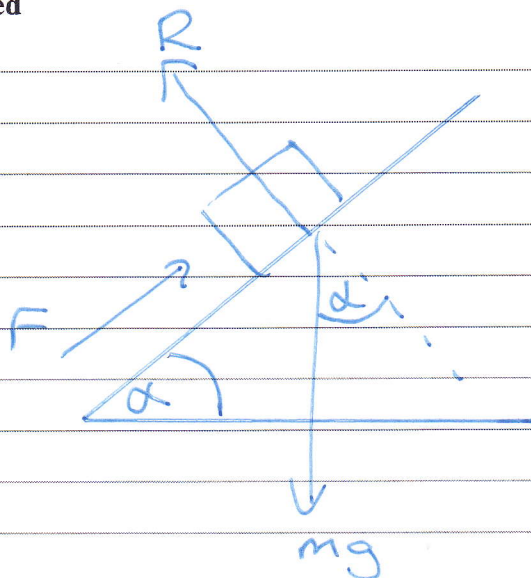
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Question 1 continued



$$\sin \alpha = \frac{3}{5}$$

$$\cos \alpha = \frac{4}{5}$$

a) R (↖) parallel to plane

$$F - mg \sin \alpha = 0 \quad (1)$$

$$F = \frac{3}{5} mg$$

R (↖) perpendicular to plane

$$R - mg \cos \alpha = 0 \quad (2)$$

$$R = \frac{4}{5} mg$$

Friction $F = \mu R$

$$b) \quad \mu = \frac{F}{R} = \frac{\frac{3}{5} mg}{\frac{4}{5} mg} = \frac{3}{4}$$

c) The forces acting on Q will still balance as the "mass" in the $F =$ and $R =$ equations will cancel each other

d) Q would slide with constant speed 0.5 m s^{-1} , no acceleration

As there is no resultant force down the plane (equilibrium), acceleration will be zero



2. A particle P moves with acceleration $(4\mathbf{i} - 5\mathbf{j})\text{ms}^{-2}$

At time $t = 0$, P is moving with velocity $(-2\mathbf{i} + 2\mathbf{j})\text{ms}^{-1}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, P passes through the origin O .

At time $t = T$ seconds, where $T > 0$, the particle P passes through the point A .

The position vector of A is $(\lambda\mathbf{i} - 4.5\mathbf{j})\text{m}$ relative to O , where λ is a constant.

(b) Find the value of T .

(4)

(c) Hence find the value of λ

(2)

$$\begin{aligned}\underline{v} &= -2\underline{i} + 2\underline{j} \text{ ms}^{-1} & \underline{v} &= \underline{u} + \underline{a}t \\ \underline{a} &= 4\underline{i} - 5\underline{j} \text{ ms}^{-2} & \underline{v} &= \begin{pmatrix} -2 \\ 2 \end{pmatrix} + 2 \begin{pmatrix} 4 \\ -5 \end{pmatrix} \\ t &= 2 & \underline{v} &= \begin{pmatrix} -2 + 8 \\ 2 - 10 \end{pmatrix} = \begin{pmatrix} 6 \\ -8 \end{pmatrix} \\ & & \underline{v} &= 6\underline{i} - 8\underline{j} \text{ ms}^{-1}\end{aligned}$$

$$\begin{aligned}\text{b) } \underline{s} &= \lambda\underline{i} - 4.5\underline{j} \\ \underline{u} &= -2\underline{i} + 2\underline{j} \\ \underline{a} &= 4\underline{i} - 5\underline{j} \\ t &= T\end{aligned}$$

$$\underline{s} = \underline{u}t + \frac{1}{2}\underline{a}t^2$$

$$\begin{pmatrix} \lambda \\ -4.5 \end{pmatrix} = T \begin{pmatrix} -2 \\ 2 \end{pmatrix} + \frac{1}{2}T^2 \begin{pmatrix} 4 \\ -5 \end{pmatrix}$$

$$\begin{aligned}\underline{i} \text{ components: } & \lambda = -2T + 2T^2 & \textcircled{1} \\ \underline{j} \text{ components: } & -4.5 = 2T - 2.5T^2 & \textcircled{2}\end{aligned}$$



Question 2 continued

② gives $2.5T^2 - 2T - 4.5 = 0$

Using calculator to solve quadratic
 $T = 1.8$ or $T = -1$

b) as $T > 0$, $T = 1.8$

c) in ①

$$\lambda = -2 \times 1.8 + 2 \times 1.8^2$$
$$\lambda = 2.88$$

(Total for Question 2 is 8 marks)



3. (i) At time t seconds, where $t \geq 0$, a particle P moves so that its acceleration \mathbf{a} ms^{-2} is given by

$$\mathbf{a} = (1 - 4t)\mathbf{i} + (3 - t^2)\mathbf{j}$$

At the instant when $t = 0$, the velocity of P is $36\mathbf{i} \text{ms}^{-1}$

- (a) Find the velocity of P when $t = 4$ (3)

- (b) Find the value of t at the instant when P is moving in a direction perpendicular to \mathbf{i} (3)

- (ii) At time t seconds, where $t \geq 0$, a particle Q moves so that its position vector \mathbf{r} metres, relative to a fixed origin O , is given by

$$\mathbf{r} = (t^2 - t)\mathbf{i} + 3t\mathbf{j}$$

Find the value of t at the instant when the speed of Q is 5ms^{-1} (6)

a)
$$\underline{\mathbf{a}} = (1 - 4t)\underline{\mathbf{i}} + (3 - t^2)\underline{\mathbf{j}}$$

integrating

$$\underline{\mathbf{v}} = (t - 2t^2)\underline{\mathbf{i}} + (3t - \frac{1}{3}t^3)\underline{\mathbf{j}} + \underline{\mathbf{c}}$$

at $t = 0$, $\underline{\mathbf{v}} = 36\underline{\mathbf{i}} \text{ms}^{-1}$

$$\underline{\mathbf{v}} = \begin{pmatrix} t - 2t^2 \\ 3t - \frac{1}{3}t^3 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 36 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\Rightarrow x = 36, y = 0$$

$$\underline{\mathbf{v}} = \begin{pmatrix} t - 2t^2 \\ 3t - \frac{1}{3}t^3 \end{pmatrix} + \begin{pmatrix} 36 \\ 0 \end{pmatrix}$$

at $t = 4$, $\underline{\mathbf{v}} = \begin{pmatrix} 4 - 2 \times 4^2 \\ 3 \times 4 - \frac{1}{3} \times 4^3 \end{pmatrix} + \begin{pmatrix} 36 \\ 0 \end{pmatrix}$

$$\underline{\mathbf{v}} = \begin{pmatrix} -28 \\ -\frac{28}{3} \end{pmatrix} + \begin{pmatrix} 36 \\ 0 \end{pmatrix} = \begin{pmatrix} 8 \\ -\frac{28}{3} \end{pmatrix}$$

$$\underline{\mathbf{v}} = 8\underline{\mathbf{i}} - \frac{28}{3}\underline{\mathbf{j}}$$



Question 3 continued

b) perpendicular to \underline{i}

when \underline{i} component of \underline{v} is zero

$$t - 2t^2 + 36 = 0$$

$$2t^2 - t + 36 = 0$$

$$(2t - 9)(t + 4) = 0$$

$$2t - 9 = 0 \quad \text{or} \quad t + 4 = 0$$

$$t = 4.5$$

$$t = -4$$

impossible

$$\therefore t = 4.5 \text{ seconds}$$

(ii)

$$\underline{r} = \begin{pmatrix} t^2 - t \\ 3t \end{pmatrix}$$

↓ differentiate

$$\underline{v} = \begin{pmatrix} 2t - 1 \\ 3 \end{pmatrix}$$

$$(2t - 1)^2 + 3^2 = 5^2$$

$$4t^2 - 4t + 1 + 9 = 25$$

$$4t^2 - 4t - 15 = 0$$

$$(2t + 3)(2t - 5) = 0$$

$$2t + 3 = 0 \quad \text{or} \quad 2t - 5 = 0$$

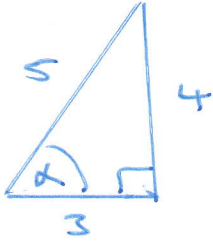
$$t = -1.5 \quad \text{or} \quad t = 2.5$$

impossible

$$\therefore t = 2.5 \text{ seconds}$$



4.



$\sin \alpha = \frac{4}{5}$
 $\cos \alpha = \frac{3}{5}$

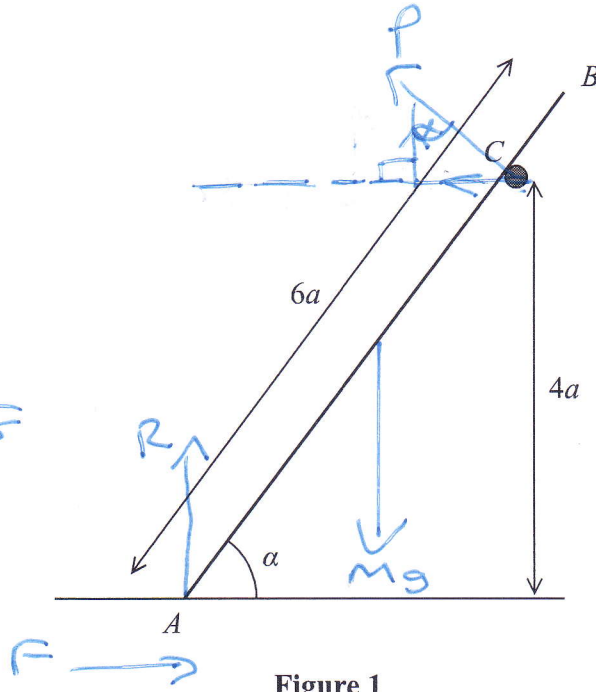
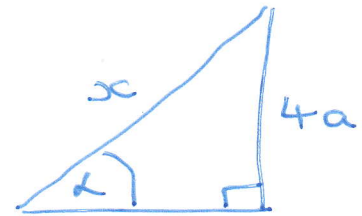


Figure 1



$\sin \alpha = \frac{4a}{x}$
 $x = \frac{4a}{\sin \alpha}$
 $x = \frac{4a}{\frac{4}{5}}$
 $x = 5a$

A ladder AB has mass M and length $6a$.

The end A of the ladder is on rough horizontal ground.

The ladder rests against a fixed smooth horizontal rail at the point C .

The point C is at a vertical height $4a$ above the ground.

The vertical plane containing AB is perpendicular to the rail.

The ladder is inclined to the horizontal at an angle α , where $\sin \alpha = \frac{4}{5}$, as shown in Figure 1.

The coefficient of friction between the ladder and the ground is μ .

The ladder rests in limiting equilibrium.

The ladder is modelled as a uniform rod.

Using the model,

(a) show that the magnitude of the force exerted on the ladder by the rail at C is $\frac{9Mg}{25}$ (3)

(b) Hence, or otherwise, find the value of μ . (7)

a) $R (\rightarrow) \quad F = P \sin \alpha \quad (1)$

$R (\uparrow) \quad R + P \cos \alpha - Mg = 0 \quad (2)$

$m(A) \quad 3a \cos \alpha \times Mg - 5a \times P = 0 \quad (3)$



Question 4 continued

$$3 \times \frac{3}{5} \times Mg = 5 \times P$$

$$\frac{9}{25} Mg = P$$

(as required)

b) in (2)

$$R = Mg - P \cos \alpha$$

$$R = Mg - \frac{9}{25} Mg \times \frac{3}{5}$$

$$R = Mg - \frac{27}{125} Mg$$

$$R = \frac{98}{125} Mg$$

(1) gives $F = P \sin \alpha$

$$F = \frac{9}{25} Mg \times \frac{4}{5}$$

$$F = \frac{36}{125} Mg$$

Limiting friction

$$F = \mu R$$

$$\mu = \frac{F}{R} = \frac{\frac{36}{125} Mg}{\frac{98}{125} Mg}$$

$$\mu = \frac{36}{98} = \frac{18}{49}$$



5.

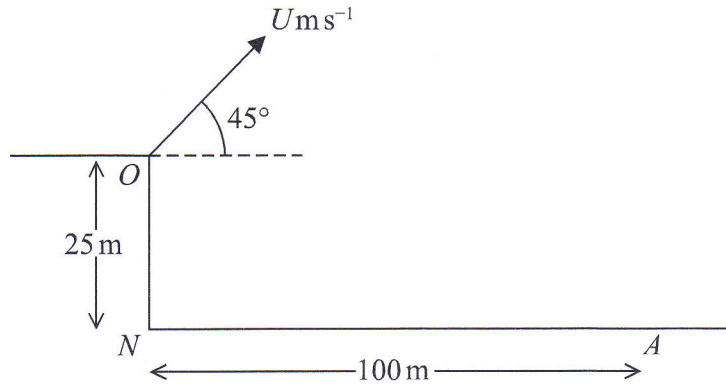


Figure 2

A small ball is projected with speed $U \text{ m s}^{-1}$ from a point O at the top of a vertical cliff.

The point O is 25 m vertically above the point N which is on horizontal ground.

The ball is projected at an angle of 45° above the horizontal.

The ball hits the ground at a point A , where $AN = 100 \text{ m}$, as shown in Figure 2.

The motion of the ball is modelled as that of a particle moving freely under gravity.

Using this initial model,

(a) show that $U = 28$ (6)

(b) find the greatest height of the ball above the horizontal ground NA . (3)

In a refinement to the model of the motion of the ball from O to A , the effect of air resistance is included.

This refined model is used to find a new value of U .

(c) How would this new value of U compare with 28, the value given in part (a)? (1)

(d) State one further refinement to the model that would make the model more realistic. (1)

a) (\rightarrow) motion $100 = U \cos 45^\circ \times t$ (1)

(\uparrow) +ve

$$s = -2.5 \text{ m}$$

$$v = U \sin 45$$

$$v =$$

$$a = -9.8 \text{ m s}^{-2}$$

$$t =$$

DO NOT WRITE IN THIS AREA



Question 5 continued

$$s = ut + \frac{1}{2} at^2$$

$$-25 = u \sin 45 \times t + \frac{1}{2} (-9.8) t^2 \quad (2)$$

(1) gives $u = \frac{100}{\cos 45 \times t}$

in (2)

$$-25 = \frac{100}{t} \times \frac{\sin 45 \times t}{\cos 45} - 4.9 t^2$$

$$-25 = 100 - 4.9 t^2$$

$$4.9 t^2 - 125 = 0$$

$$t = 5.050762723, -5.050762723$$

in (1) $u = \frac{100}{5.050762723 \times \cos 45}$

$$u = 28 \quad (\text{as required})$$

B)

$$s = ?$$

$$u = 28 \sin 45 \text{ m s}^{-1}$$

$$v = 0 \text{ m s}^{-1}$$

$$a = -9.8 \text{ m s}^{-2}$$

$$t = ?$$

$$v^2 = u^2 + 2as$$

$$s = \frac{v^2 - u^2}{2a} = \frac{0^2 - (28 \sin 45)^2}{2 \times -9.8}$$

$$s = 20 \text{ m}$$

$$\text{Height above ground} = 20 + 25 = 45 \text{ m}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



Question 5 continued

c) To reach point A the new value of u would have to be larger than 28 to compensate for wind resistance.

d) Account for effects of wind
or
Use more accurate value for g
or
Account for spin of the ball

