Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Monday 18 October 2021 – Afternoon

Paper reference **9MA0/32**

Mathematics

Advanced
PAPER 32: Mechanics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 50. There are 5 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

1. A particle P moves with constant acceleration $(2\mathbf{i} - 3\mathbf{j})\,\mathrm{m\,s^{-2}}$

At time $t = 0$, P is moving with velocity $4\mathbf{i}\,\mathrm{m\,s^{-1}}$

(a) Find the velocity of P at time $t = 2$ seconds.

(2)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})\,\mathrm{m}$.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

a) $\underset{\sim}{a} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

integrating $\underset{\sim}{v} = \begin{pmatrix} 2t \\ -3t \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$

at $t = 0$, $\underset{\sim}{v} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$\begin{pmatrix} 2t \\ -3t \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$

$a = 4$

$b = 0$

$\therefore \quad \underset{\sim}{v} = \begin{pmatrix} 2t+4 \\ -3t \end{pmatrix}$

at $t = 2$, $\underset{\sim}{v} = \begin{pmatrix} 2\times 2+4 \\ -3\times 2 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$

b)

At time $t = 0$, the position vector of P relative to a fixed origin O is $(\mathbf{i} + \mathbf{j})$ m.

(b) Find the position vector of P relative to O at time $t = 3$ seconds.

(2)

integrating $\underset{\sim}{v}$: $\underset{\sim}{s} = \begin{pmatrix} t^2+4t \\ -\frac{3}{2}t^2 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$

at $t = 0$ $\quad \underset{\sim}{s} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \therefore \quad a = 1, \; b = 1$

$\underset{\sim}{s} = \begin{pmatrix} t^2+4t+1 \\ -\frac{3}{2}t^2+1 \end{pmatrix} \quad \therefore$ at $t = 3$

$\underset{\sim}{s} = \begin{pmatrix} 9+12+1 \\ -\frac{27}{2}+1 \end{pmatrix} = \begin{pmatrix} 22 \\ -12.5 \end{pmatrix}$

2.

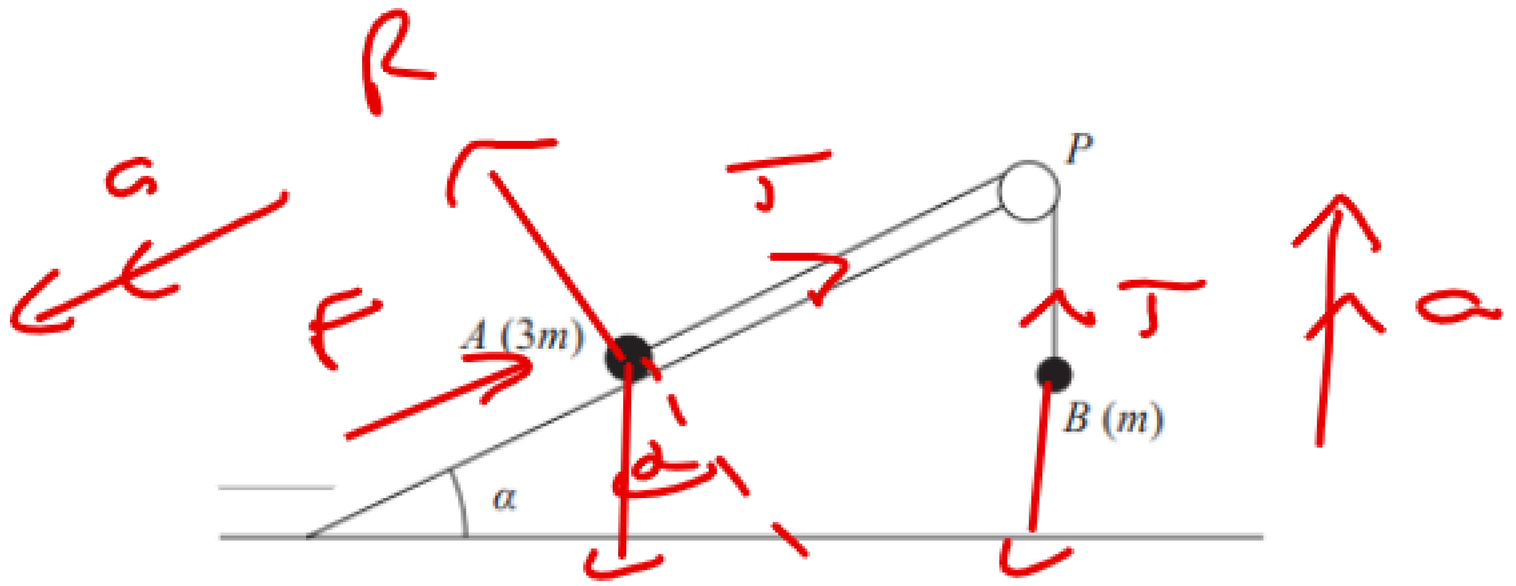


Figure 1

A small stone A of mass $3m$ is attached to one end of a string.

A small stone B of mass m is attached to the other end of the string.

Initially A is held at rest on a fixed rough plane.

The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

The string passes over a pulley P that is fixed at the top of the plane.

The part of the string from A to P is parallel to a line of greatest slope of the plane.

Stone B hangs freely below P, as shown in Figure 1.

The coefficient of friction between A and the plane is $\frac{1}{6}$

Stone A is released from rest and begins to move down the plane.

The stones are modelled as particles.

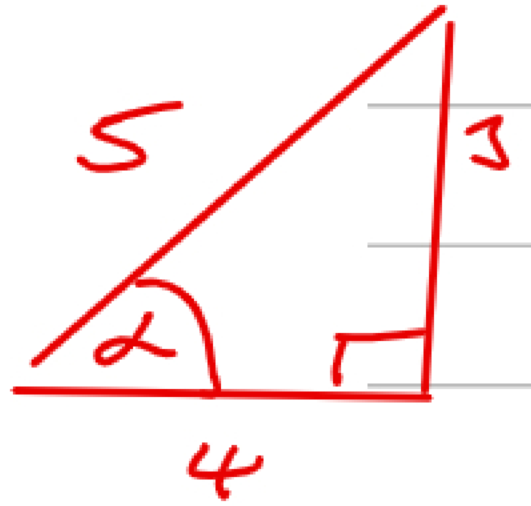
The pulley is modelled as being small and smooth.

The string is modelled as being light and inextensible.

Using the model for the motion of the system before B reaches the pulley,

(a) write down an equation of motion for A

(2)



$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$\mu = \frac{1}{6}$

a) motion for A parallel to plane

$3m \times a = 3mg\sin\alpha - F - T$

$3ma = 3mg \times \frac{3}{5} - F - T$ (1)

(b) show that the acceleration of A is $\frac{1}{10}g$

(7)

b) motion for B (T)

$m \times a = T - mg$ (2)

Friction $F = \mu R$

$F = \frac{1}{6}R$ (3)

motion A perpendicular to plane

$0 = R - 3mg\cos\alpha$

$R = 3mg \times \frac{4}{5} = \frac{12}{5}mg$ (4)

in (3) $F = \frac{1}{6} \times \frac{12}{5}mg$

$F = \frac{2}{5}mg$

in (1) $\quad 3ma = 3mg \times \frac{3}{5} - F - T \quad (1)$

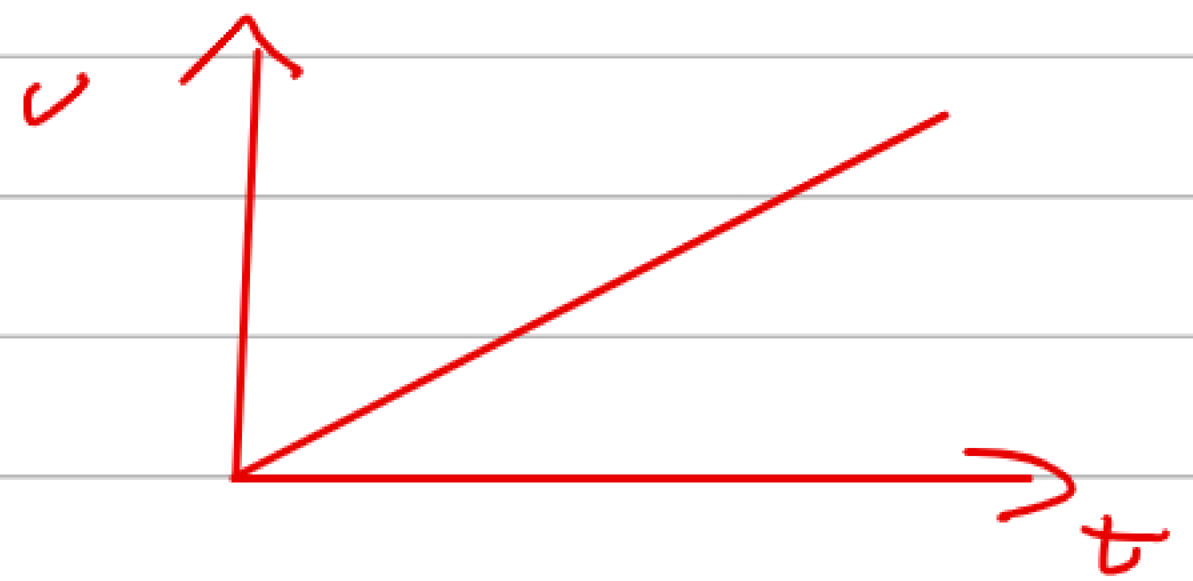
$$3ma = \frac{9}{5}mg - \frac{2}{5}mg - T$$

(2) gives $T = ma + mg$

$$\therefore \quad 3ma = \frac{9}{5}mg - \frac{2}{5}mg - ma - mg$$

$$4\cancel{m}a = \frac{2}{5}\cancel{m}g$$

$$a = \frac{2}{20}g = \frac{1}{10}g \quad \text{(as required)}$$

2c) 

d) Tension in A would be different to tension in B

3.

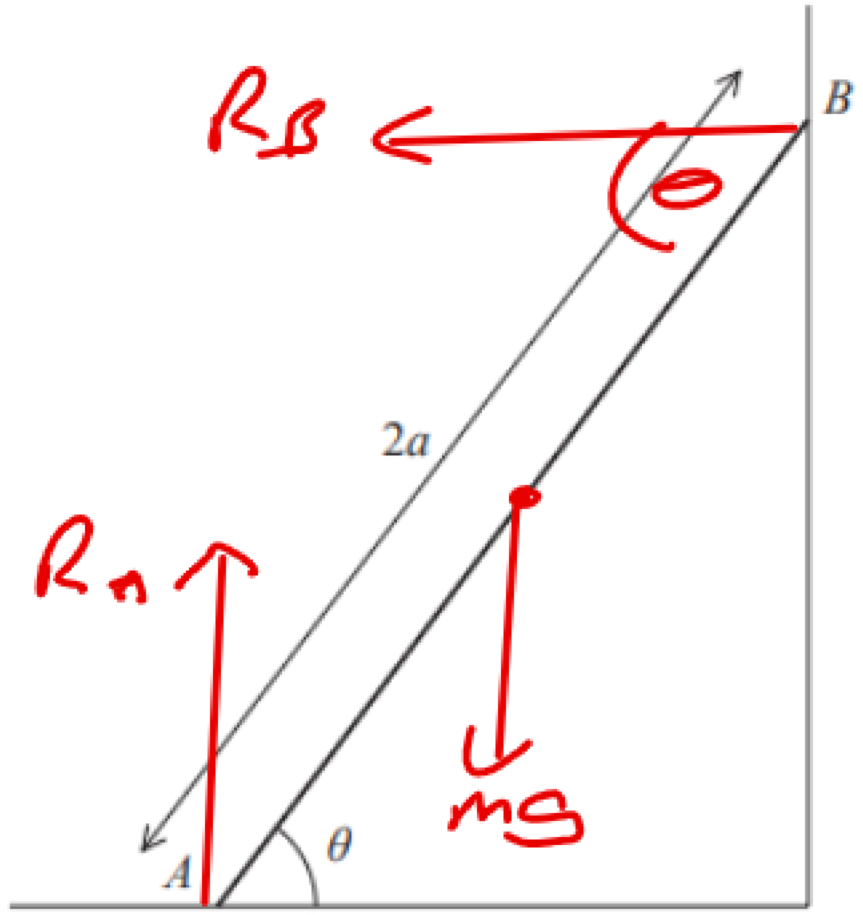


Figure 2

F

A beam AB has mass m and length $2a$.

The beam rests in equilibrium with A on rough horizontal ground and with B against a smooth vertical wall.

The beam is inclined to the horizontal at an angle θ, as shown in Figure 2.

The coefficient of friction between the beam and the ground is μ

The beam is modelled as a uniform rod resting in a vertical plane that is perpendicular to the wall.

Using the model,

(a) show that $\mu \geqslant \frac{1}{2}\cot\theta$

(5)

a) $R(\uparrow)$

$R_A = mg$ ①

$R(\rightarrow)$

$F = R_B$ ②

Friction

$F \le \mu R_A$ ③

$M(A)$ $\cancel{a}\cos\theta \times mg = 2\cancel{a}\sin\theta \times R_B$

$\dfrac{mg \times \cos\theta}{2 \times \sin\theta} = R_B$ ④

③ gives $F \le \mu \times mg$

$\dfrac{\cancel{mg}}{2} \times \cot\theta \le \mu \times \cancel{mg}$

$\therefore \mu \geqslant \frac{1}{2}\cot\theta$

as required

b)

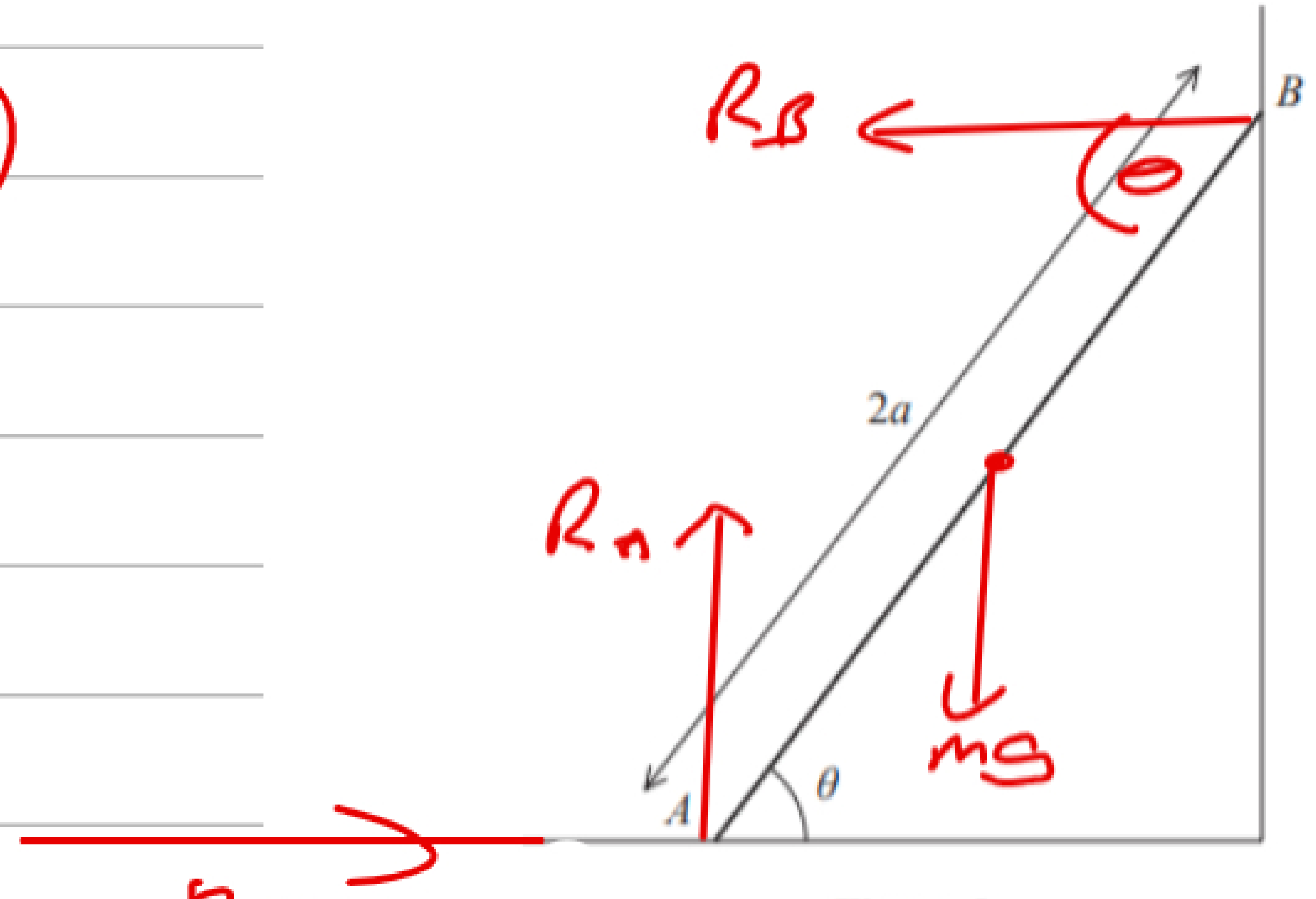


Figure 2

F

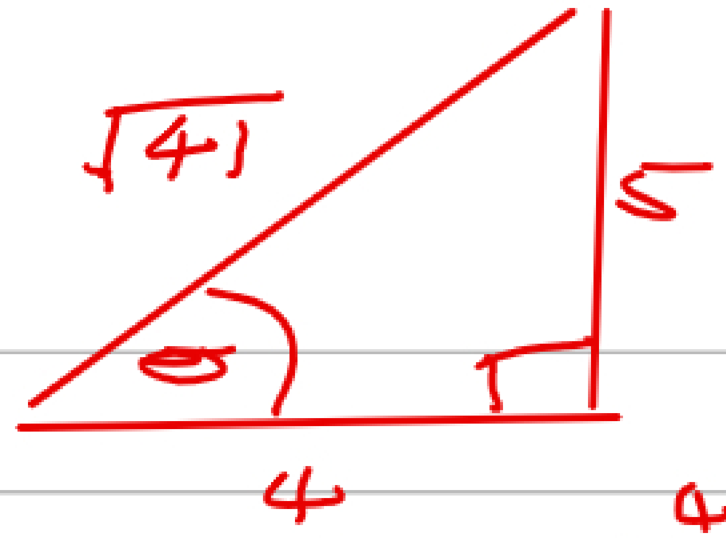
A horizontal force of magnitude *kmg*, where *k* is a constant, is now applied to the beam at *A*.

This force acts in a direction that is perpendicular to the wall and towards the wall.

Given that $\tan\theta = \frac{5}{4}$, $\mu = \frac{1}{2}$ and the beam is now in limiting equilibrium,

(b) use the model to find the value of *k*.

(5)



$\tan\theta = \frac{5}{4}$

$\sin\theta = \frac{5}{\sqrt{41}}$ $\quad \cos\theta = \frac{4}{\sqrt{41}}$ $\quad \mu = \frac{1}{2}$

$R(\uparrow)$ $\quad R_A = mg$ (1)

$R(\rightarrow)$ $\quad kmg = F + R_B$ (2)

$M(B)$ $\quad a\cos\theta \times mg + 2a\sin\theta \times kmg = 2a\cos\theta \times R_A + 2a\sin\theta \times F$

$$\frac{4}{\sqrt{41}} a\,mg + 2a \times \frac{5}{\sqrt{41}} \times kmg = 2a \times \frac{4}{\sqrt{41}} \times mg + 2a \times \frac{5}{\sqrt{41}} \times F \quad (3)$$

Limiting friction $\quad F = \mu R_A$ $\quad$ in (3)

$F = \frac{1}{2} mg$

$$\frac{4}{\sqrt{41}} \cancel{a}\, \cancel{m} \cancel{g} + 2\cancel{a} \times \frac{5}{\sqrt{41}} \times k \cancel{m} \cancel{g} = 2\cancel{a} \times \frac{4}{\sqrt{41}} \times \cancel{m} \cancel{g} + 2\cancel{a} \times \frac{5}{\sqrt{41}} \times \frac{1}{2} \cancel{m} \cancel{g}$$

$$\frac{4}{\sqrt{41}} + \frac{10k}{\sqrt{41}} = \frac{8}{\sqrt{41}} + \frac{5}{\sqrt{41}}$$

numerators $\therefore$ $4 + 10k = 13$

$$10k = 9$$

$$k = 0.9$$

4.

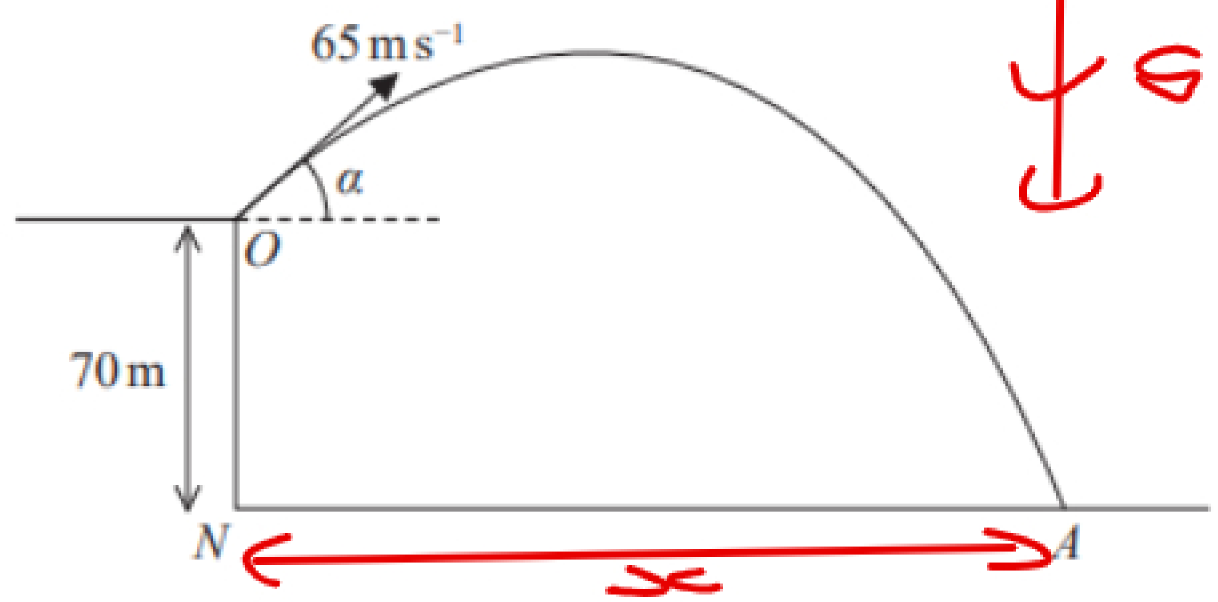


Figure 3

↓ g

a) Horizontal motion (→)

$x = 65\cos\alpha \times t$

$x = 65 \times \frac{12}{13} \times t = 60t$ (1)

Vertical motion (↑) +ve

$s = -70$

$u = 65\sin\alpha = 65 \times \frac{5}{13} = 25$

$v = ?$

$a = -10$

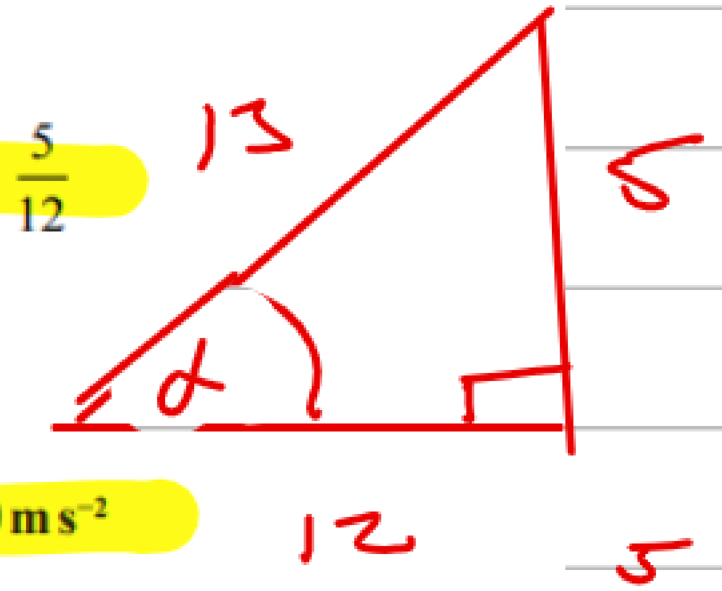
$t = ?$

A small stone is projected with speed $65\,\mathrm{m\,s^{-1}}$ from a point O at the top of a vertical cliff.

Point O is 70 m vertically above the point N.

Point N is on horizontal ground.

The stone is projected at an angle α above the horizontal, where $\tan\alpha = \frac{5}{12}$



The stone hits the ground at the point A, as shown in Figure 3.

The stone is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude $10\,\mathrm{m\,s^{-2}}$

$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$

$s = ut + \frac{1}{2}at^2$

$-70 = 25t - 4.9t^2$

$5t^2 - 25t - 70 = 0$

equation solver on Classwiz

$t = 7$ $t = -2$

$\therefore t = 7$ seconds

Using the model,

(a) find the time taken for the stone to travel from O to A,

(4)

(b) find the speed of the stone at the instant just before it hits the ground at A.

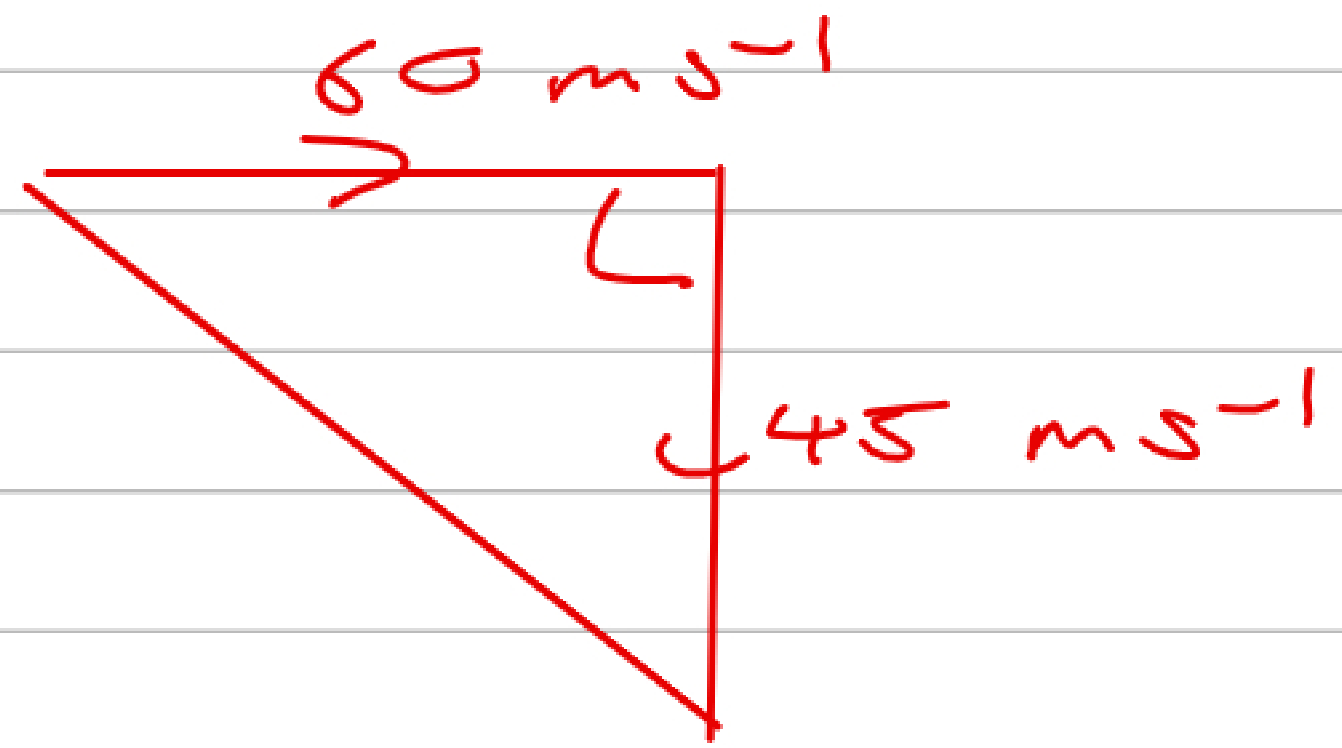
(5)

b) at $t = 7$ $\qquad$ $v = u + at$

Horizontal velocity $\qquad$ $v = 25 - 10 \times 7 = -45\ ms^{-1}$

$65 \cos\alpha$ $\qquad$ ↑ vertical velocity

$= 65 \times \frac{12}{13} = 60$



speed at A

$= \sqrt{60^2 + 45^2}$

$= 75\ ms^{-1}$

c) used approximate value $g = 10\ ms^{-2}$ instead of $g = 9.8\ ms^{-2}$

3. At time t seconds, a particle P has velocity $\mathbf{v}$ m s^{-1}, where

$$\mathbf{v} = 3t^{\frac{1}{2}}\,\mathbf{i} - 2t\,\mathbf{j} \qquad t > 0$$

(a) Find the acceleration of P at time t seconds, where $t > 0$ **(2)**

(b) Find the value of t at the instant when P is moving in the direction of $\mathbf{i} - \mathbf{j}$ **(3)**

At time t seconds, where $t > 0$, the position vector of P, relative to a fixed origin O, is $\mathbf{r}$ metres.

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t. **(3)**

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\text{m s}^{-1}$ **(6)**

a) $\underset{\sim}{v} = \begin{pmatrix} 3t^{\frac{1}{2}} \\ 2t \end{pmatrix}$

differentiate

$\underset{\sim}{a} = \dfrac{dv}{dt} = \begin{pmatrix} \frac{3}{2} t^{-\frac{1}{2}} \\ 2 \end{pmatrix}$

b) set $\underset{\sim}{i}$ component equal to $\underset{\sim}{j}$ component

$3t^{\frac{1}{2}} = 2t$

$3t^{\frac{1}{2}} - 2t = 0$

$t^{\frac{1}{2}}(3 - 2t^{\frac{1}{2}}) = 0$

$t = 0$ or $2\sqrt{t} = 3$

$t = \left(\frac{3}{2}\right)^2$

$t = \frac{9}{4}$

When $t = 1$, $\mathbf{r} = -\mathbf{j}$

(c) Find an expression for $\mathbf{r}$ in terms of t.

(3)

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\text{m s}^{-1}$

(6)

$$\underset{\sim}{v} = \begin{pmatrix} 3t^{\frac{1}{2}} \\ -2t \end{pmatrix}$$

integrate

$$\underset{\sim}{r} = \begin{pmatrix} \frac{2\times 3}{3} t^{\frac{3}{2}} \\ -\frac{2t^2}{2} \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\underset{\sim}{r} = \begin{pmatrix} 2t^{\frac{3}{2}} \\ -t^2 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

$$t = 1, \quad r = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$$\underset{\sim}{r} = \begin{pmatrix} 2 \\ -1 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} \Rightarrow \quad a = -2, \quad b = 0$$

$$\underset{\sim}{r} = \begin{pmatrix} 2t^{\frac{3}{2}} - 2 \\ -t^2 \end{pmatrix}$$

(d) Find the exact distance of P from O at the instant when P is moving with speed $10\,\mathrm{m\,s^{-1}}$

(6)

$$\underset{\sim}{v} = \begin{pmatrix} 3t^{\frac{1}{2}} \\ -2t \end{pmatrix}$$

$$\text{speed} = 10 = \sqrt{\left(3t^{\frac{1}{2}}\right)^2 + (-2t)^2}$$

$$10 = \sqrt{9t + 4t^2}$$

square both sides

$$100 = 9t + 4t^2$$

$$0 = 4t^2 + 9t - 100$$

Equation solver on calculator

$$t = 4, \quad t = \frac{-25}{4}$$

$$\therefore t = 4$$

$$\underset{\sim}{r} = \begin{pmatrix} 2t^{\frac{3}{2}} - 2 \\ -t^2 \end{pmatrix} \quad \text{when } t = 4$$

$$\underset{\sim}{r} = \begin{pmatrix} 2 \times 4^{\frac{3}{2}} - 2 \\ -16 \end{pmatrix} = \begin{pmatrix} 14 \\ -16 \end{pmatrix}$$

$$\text{distance from } O = \sqrt{14^2 + (-16)^2}$$
$$= 2\sqrt{113} \text{ m}$$