1 Solve $3^x = 13$, giving your answer to 3 significant figures.

$x = \log_3 13$

$x = 2.33$

2 Solve $2^x = 32$

$x = 5$

3 Solve the equation

$$2\log_2(x) - \log_2(5) = 1$$

$\log_2 x^2 - \log_2 5 = 1$

$\log_2\left(\frac{x^2}{5}\right) = 1$

$2^1 = \frac{x^2}{5}$

$2 = \frac{x^2}{5}$

$x^2 = 10$

$x = \sqrt{10}$

4 Solve the equation

$$\log_3(x) + \log_3(4) = 2$$

$\log_3 4x = 2$

$3^2 = 4x$

$9 = 4x$

$x = \frac{9}{4}$

5 Express as a single logarithm to base a

$$2\log_a(x+1) - \log_a(4)$$

$$\log_a(x+1)^2 - \log_a 4$$

$$\log_a\left(\frac{(x+1)^2}{4}\right)$$

---

6 Giving your answers to 2 decimal places, solve the simultaneous equations

$$e^{2y} = x + 1$$
$$\ln(x-2) = 2y - 1$$

$$\ln(x-2) = 2y - 1$$

$$x - 2 = e^{2y-1}$$

$$x - 2 = \frac{e^{2y}}{e}$$

$$ex - 2e = e^{2y} \quad (1) \qquad e^{2y} = x + 1 \quad (2)$$

$$ex - 2e = x + 1$$

$$ex - x = 2e + 1$$

$$x(e-1) = 2e + 1$$

$$x = \frac{2e+1}{e-1}$$

$$= \underline{3.75}$$

$$e^{2y} = 3.75 + 1$$

$$2y = \ln(4.75)$$

$$y = \underline{0.779}$$

7 Solve the equation

$$\ln(2x + 5) = 1$$

$$2x + 5 = e$$

$$x = \frac{e - 5}{2}$$

8 Given that $y = \log_2 x$, find expressions in terms of y for

(a) $\log_2 x^2$

(b) $\log_2 2x$

(c) $\log_8 x$

a/ $2 \log_2 x$

$2y$

b/ $\log_2 x + \log_2 2$

$\log_2 x + 1$

$y + 1$

c/ $\log_8 x = \frac{\log_2 x}{\log_2 8}$

$= \frac{\log_2 x}{3}$

$\frac{y}{3}$

9 Solve the equation, giving your answers in exact form.

$$2e^{y} + 15e^{-y} = 11$$

$$2e^{2y} + 15 = 11e^{y}$$

$$2e^{2y} - 11e^{y} + 15 = 0$$

$$(2e^{y} - 5)(e^{y} - 3) = 0$$

$$e^{y} = \frac{5}{2} \qquad e^{y} = 3$$

$$y = \ln\left(\frac{5}{2}\right) \qquad y = \ln 3$$

---

10 The population of a species of plant in a field is modelled using the formula $P = 50e^{0.1t}$
Where t is the number of weeks since the population was first recorded.

(a) Write down the number of the plants when the population was first recorded. **(1)**

(b) Find the rate of increase in the population 10 weeks after the population was first recorded. **(2)**

(c) Find how many weeks it takes for the number of plants to exceed 300. **(4)**

a/ 50

b/ $\frac{dP}{dt} = 5e^{0.1t}$

when $t = 10$ $\qquad \frac{dP}{dt} = 5e$

c/ $300 = 50e^{0.1t}$

$6 = e^{0.1t}$

$\ln 6 = 0.1t$

$10 \ln 6 = t$

$t = 17.9$ $\therefore$ 18 weeks

11 The decay of a radioactive substance is modelled using the formula $N = 1000e^{-kt}$
Where N is the number of atoms after t years and k is a positive constant.

(a) Write down the number of atoms when the substance started to decay. **(1)**

Given it takes 14.4 years for half of the substance to decay.

(b) Find the value of k to three significant figures. **(4)**

(c) Calculate the number of atoms left when t=30. **(1)**

a/ 1000

b/ $500 = 1000e^{-14.4k}$

$\frac{1}{2} = e^{-14.4k}$

$\ln\left(\frac{1}{2}\right) = -14.4k$

$k = 0.0481$

c/ $N = 1000e^{-0.0481(30)}$

$= \underline{\underline{236}}$

---

12 The temperature of water in a kettle is modelled using the formula $T = 75e^{-kt} + 22$

Where T is the temperature t minutes after the kettle is turned off and k is a positive constant.

(a) Find the rate of change of the temperature in terms of k **(2)**

After 5 minutes the temperature of the water is 70°C

(b) Find the value of k **(3)**

(c) Find how many minutes it takes for the water to cool to 55°C **(4)**

a/ $\frac{dT}{dt} = \underline{\underline{-75ke^{-kt}}}$

b/ $70 = 75e^{-5k} + 22$

$\frac{16}{25} = e^{-5k}$

$\ln\left(\frac{16}{25}\right) = -5k$

$\underline{\underline{k = 0.0893}}$

c/ $55 = 75e^{-0.0893t} + 22$

$\frac{11}{25} = e^{-0.0893t}$

$\ln\frac{11}{25} = -0.0893t$

$t = 9.20$

<u>9 minutes to the nearest minute.</u>

13 $f(x) = e^{2x+1} - 3$

(a) State the range of f **(1)**

The curve y = f(x) meets the *y*-axis at *A* and the *x*-axis at *B*.

(b) Find the exact coordinates of *A* and *B*. **(4)**

(c) Find the equation of the tangent to the curve at *A*. **(4)**

a/ $f(x) > -3$

b/ $y = e^{2x+1} - 3$

crosses y when $x = 0$ $y = \underline{e - 3}$

$A: (0, e-3)$

x_1 y_1

crosses x when $y = 0$

$$0 = e^{2x+1} - 3$$
$$3 = e^{2x+1}$$
$$\ln 3 = 2x - 1$$
$$\frac{\ln(3) + 1}{2} = x$$

$B: \left(\frac{\ln(3)+1}{2}, 0\right)$

c/ $f'(x) = 2e^{2x+1}$

$f'(0) = 2e$ M

$y - (e-3) = 2e(x)$

$y - e + 3 = 2ex$

$\underline{y = 2ex + e - 3}$

14 The population of bacteria is being measured.

The equation

$$\log_{10}P = 0.5t + 1.398$$

is used to model the population of bacteria, P, t hours after it was first recorded.

(a) Show that $P = ab^t$, where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures. **(4)**

(b) Interpret the meaning of the constant a in this model. **(1)**

(c) Find the population of the bacteria after 10 hours. Give your answer to 2 significant figures. **(2)**

a/ $P = 10^{0.5t + 1.398}$

$= 10^{0.5t} \cdot 10^{1.398}$

$= 10^{1.398} (10^{0.5})^t$

$a = 10^{1.398}$ $\quad$ $b = 10^{0.5}$

$= \underline{\underline{25}}$ $\quad$ $= \underline{\underline{3.16}}$

b/ The initial population of bacteria

c/ $P = 25 \cdot 3.16^{10}$

$= \underline{\underline{2500000}}$

15 The growth in the population of worms, W, is modelled by the equation:

$$W = 95 - 75e^{kt}$$

Where k is a constant and t is the the number of days since the first measurement.

(a) Use the model to find the number of worms when measurements began. **(1)**

After 50 days there were 35 worms

(b) Use this information to find a complete equation for the model, giving your value of k to 3 significant figures. **(4)**

(c) Use the model to predict the number of worms after one year. **(1)**

(d) Sketch the graph of W against t. **(3)**

a/ 20

b/ $35 = 95 - 75e^{50k}$

$\frac{4}{5} = e^{50k}$

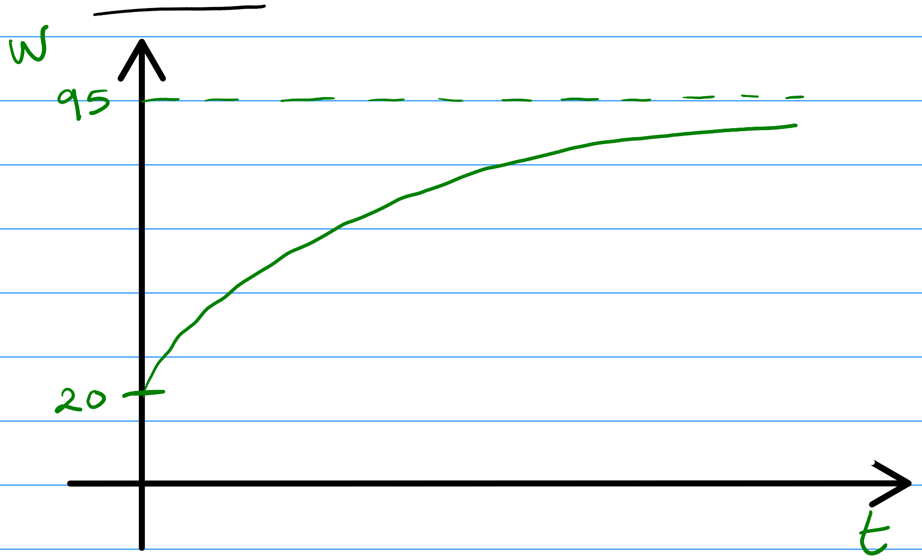
$\ln\left(\frac{4}{5}\right) = 50k$

$k = -0.00446$

$$\underline{W = 95 - 75e^{-0.00446t}}$$

c/ $W = 95 - 75e^{-0.00446(365)}$

$= \underline{80}$



16 Two experiments measuring the population of insects are started at the same time.

The number of insects in experiment A is modelled by the formula $P_A = ae^{0.2t}$

The number of insects in experiment B is modelled by the formula $P_B = be^{0.15t}$

Where t is the number of days since the experiments began.

At the start of the experiments the **total** number of insects recorded is 120
After 8 days the **total** number of insects recorded is 480

(a) Show that $a = 50$, to the nearest whole number, and find the value of b. **(3)**

(b) Estimate the total number of insects after 10 days. **(1)**

(c) Find the day that the number of insects in experiment A exceed the number of insects in experiment B. **(3)**

a)
$$a + b = 120 \quad (1)$$
$$ae^{0.2(8)} + be^{0.15(8)} = 480$$
$$ae^{1.6} + be^{1.2} = 480 \quad (2)$$

$$a = 49.96 = \underline{\underline{50}} \qquad b = 70.036\ldots = \underline{\underline{70}}$$

b)
$$50e^{0.2(10)} + 70e^{0.15(10)} = \underline{\underline{683}}$$

c)
$$50e^{0.2t} > 70e^{0.15t}$$
$$5e^{0.05t} > 7$$
$$e^{0.05t} > \tfrac{7}{5}$$
$$0.05t > \ln\left(\tfrac{7}{5}\right)$$
$$t > 20\ln\left(\tfrac{7}{5}\right)$$
$$t > 6.73$$

$\underline{7 \text{ days}}$

17 The temperature, θ°C, of a cup of tea t minutes after measurements began, is modelled using the formula

$$\theta = 68e^{-0.15t} + 21 \qquad t \geq 0$$

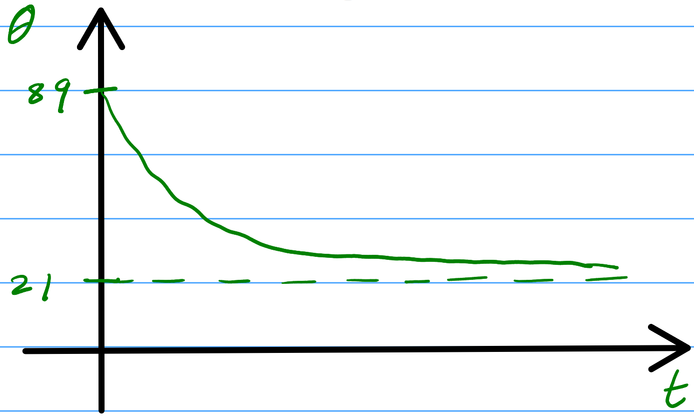
(a) Sketch the graph of θ against t. **(2)**

Find according to the model:

(b) the initial temperature of the tea. **(1)**

(c) Find the value of t when the cup of tea reaches 40°C. **(3)**

a/



b/ 89°

c/ $40 = 68e^{-0.15t} + 21$

$\frac{19}{68} = e^{-0.15t}$

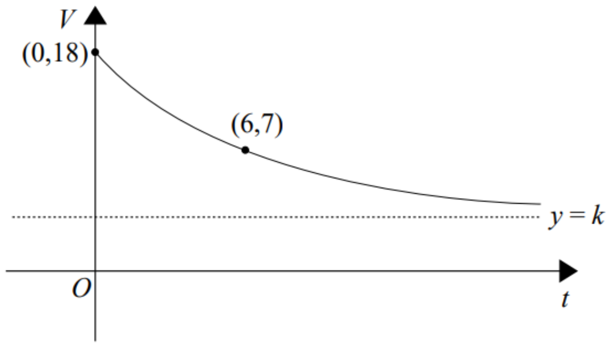
$\ln\left(\frac{19}{68}\right) = -0.15t$

$t = 8.50$ mins

18 The value of a car t years after it was purchased, where V is the value of the car in thousands of pounds, is modelled by the formula

$$V = A + Be^{\frac{-1}{6}t}$$

where A and B are constants.



The graph shows a sketch of V against t. The graphs passes through (0, 18) and (6, 7).
$y = k$ is an asymptote to the curve, where k is a constant.

Find the values of A, B and k.

$A + B = 18$ ①

$7 = A + Be^{-1}$

$A + e^{-1}B = 7$ ②

$A = 0.598$ $B = 17.4$

$V = 0.598 + 17.4e^{-\frac{1}{6}t}$

$k = 0.598$

---

19 Show that

$$\log_2\left(\frac{x}{8}\right) + 2\log_2 3 \equiv \log_2(9x) - 3$$

$\log_2 x - \log_2 8 + \log_2 3^2$

$\log_2 x - 3 + \log_2 9$

$\log_2 x + \log_2 9 - 3$

$\log_2(9x) - 3$

20 (a) Using $y = 2^x$ as a substitution, show that

$$4^x - 2^{x+2} - 5 = 0$$

can be written as

$$y^2 - 4y - 5 = 0$$ **(2)**

(b) Hence, show that the equation

$$4^x - 2^{x+2} - 5 = 0$$

has $x = \log_2 5$ as its only solution. **(4)**

a/
$$2^{2x} - 2^x \cdot 2^2 - 5 = 0$$
$$(2^x)^2 - 4 \cdot 2^x - 5 = 0$$
$$y^2 - 4y - 5 = 0$$

b/
$$(y-5)(y+1) = 0$$
$$y = 5 \qquad y = -1$$
$$2^x = 5 \qquad 2^x = -1$$
$\underline{x = \log_2 5}$ no sol, 2^x cannot be negative

---

21 A curve has the equation $y = e^{2x}$

At point P on the curve the tangent is parallel to the line $9x - 2y + 3 = 0$

Find the coordinates of P stating your answer in the form $(\ln p, q)$, where p and q are rational.

$$2y = 9x + 3$$
$$y = \frac{9}{2}x + \frac{3}{2} \qquad \underline{m = \frac{9}{2}}$$

$$\frac{dy}{dx} = 2e^{2x}$$

$$2e^{2x} = \frac{9}{2}$$

$$e^{2x} = \frac{9}{4}$$

$$2x = \ln\frac{9}{4}$$

$$x = \frac{1}{2}\ln\frac{9}{4} = \ln\left(\frac{3}{2}\right)$$

$$y = e^{2\ln\left(\frac{3}{2}\right)} = \frac{9}{4} \qquad \underline{\left(\ln\frac{3}{2}, \frac{9}{4}\right)}$$

22 The population of people in a town is modelled using the formula

$$P = a(10^{bt})$$

where t is the time in years since 2001, and a and b are constants.

(a) Explain what the value of a represents **(1)**

In 2008 the population of the town was 54 000
In 2013 the population of the town was 59 000

(b) Use the data to calculate the value of a and the value of b. **(4)**

a/ a is the population in 2001

b/ $54000 = a(10^{7b})$ $\qquad 59000 = a(10^{12b})$

$$\frac{59000}{54000} = \frac{a(10^{12b})}{a(10^{7b})}$$

$$\frac{59}{54} = 10^{5b}$$

$$5b = \log_{10}\left(\frac{59}{54}\right)$$

$$\underline{\underline{b = 0.00769}}$$

$$\frac{54000}{10^{7(0.00769)}} = a$$

$$\underline{\underline{a = 47700 \ (3sf)}}$$

---

23 Find the solution to

$$5^{2x} = 9$$

giving your answer in the form $\log_5 a$, where a in an integer.

$$2x = \log_5 9$$

$$x = \tfrac{1}{2}\log_5 9$$

$$= \log_5 9^{\frac{1}{2}}$$

$$= \underline{\underline{\log_5 3}}$$

24 A curve has the equation $y = e^{2x}$

(a) Find, in terms of a, the equation of the tangent to the curve at the point (a, e^{2a}) **(3)**

(b) Find the value of a for which this tangent passes through the origin. **(2)**

(c) Hence, find the set of values of m for which the equation

$$e^{2x} = mx$$

has no real solutions. **(3)**

a/ $\frac{dy}{dx} = 2e^{2x}$

$m = 2e^{2a}$

$y - e^{2a} = 2e^{2a}(x - a)$

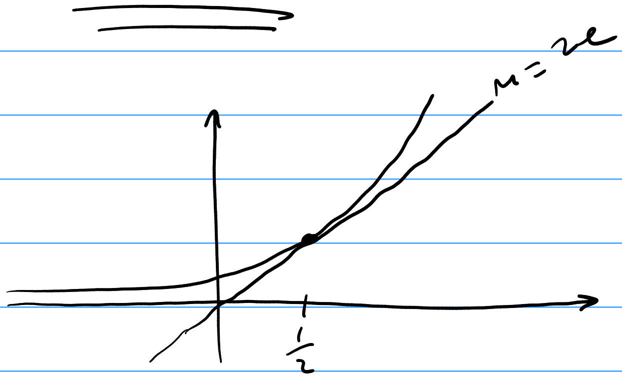
$y = 2e^{2a}x - 2ae^{2a} + e^{2a}$

b/ $e^{2a} - 2ae^{2a} = 0$

$e^{2a}(1 - 2a) = 0$

$\underline{\underline{a = \frac{1}{2}}}$

c/



$0 \leq m < 2e$

25 Jonathan is investigating the spread of a virus measured by the number of daily recorded cases N.

He believes that V and t are connected by a formula:

$$N = Ae^{kt}$$

where t is the number of days since the virus was first recorded and where A and k are constants.

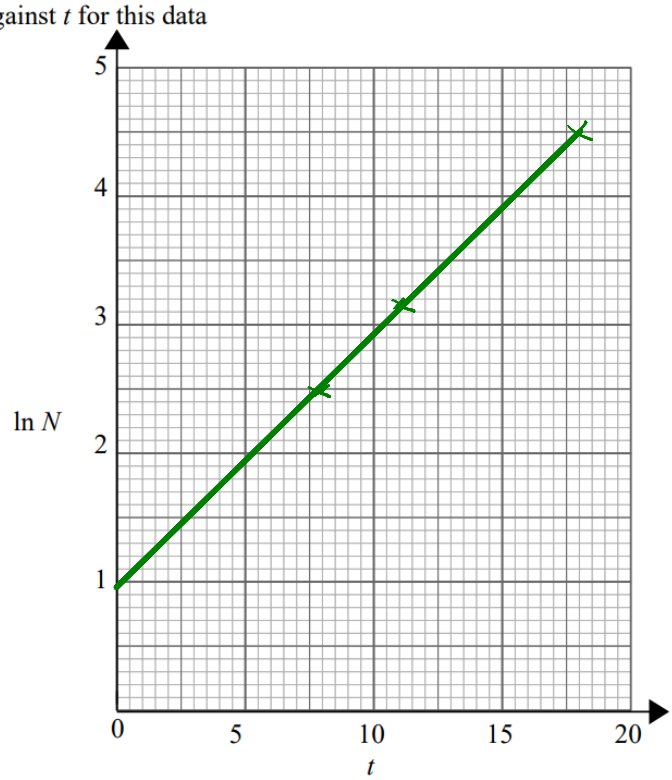
(a) Express ln N in terms of t

(2)

Jonathan collects the following data

t	8	11	18
N	12	24	90

(b) Plot ln N against t for this data



(2)

(c) Using the graph, estimate the value of A and the value of k.

(4)

a/ $N = Ae^{kt}$

$\ln N = \ln(Ae^{kt})$

$= \ln A + \ln e^{kt}$

$\ln N = \ln A + kt$

c/ $\ln A = 1$

$A = e$

$k = \dfrac{3.4}{17.5} = \underline{\underline{0.19}}$ 2sf

26 The temperature of water in a kettle is modelled using the equation $T = 75e^{-kt} + 25$

Where T is the temperature t minutes after the kettle is turned off and k is a positive constant.

(a) Explain what the 25 represents in the equation $T = 75e^{-kt} + 25$ **(1)**

When the kettle is turned off the rate of decrease of the water temperature is 20°C per minute.

(b) Find the value of k **(3)**

(c) Find how many minutes it takes for the water to cool to 55°C **(3)**

a) The room temperature, the temperature the water approaches.

b) $$\frac{dT}{dt} = -75ke^{-kt}$$

$$-75k = -20$$

$$k = \frac{4}{15}$$

c) $$55 = 75e^{-\frac{4}{15}t} + 25$$

$$\frac{2}{5} = e^{-\frac{4}{15}t}$$

$$\ln\left(\frac{2}{5}\right) = -\frac{4}{15}t$$

$$t = 3.44 \text{ mins}$$

27 The line L is a tangent to the curve $y = e^{\frac{1}{3}x}$ at the point where $x = 3$

Show that L passes through the origin.

$(3, e)$ $\frac{dy}{dx} = \frac{1}{3}e^{\frac{1}{3}x}$

when $x = 3$ $\frac{dy}{dx} = \frac{1}{3}e$

$y - e = \frac{1}{3}e(x - 3)$

$y - e = \frac{1}{3}ex - e$

$y = \frac{1}{3}ex$

$c = 0$ $\therefore$ passes through x axis.

28 Find the coordinates of the point of intersection of the curves $y = e^x$ and $y = 3 - 2e^{\frac{1}{2}x}$

$e^x = 3 - 2e^{\frac{1}{2}x}$

$e^x + 2e^{\frac{1}{2}x} - 3 = 0$

$(e^{\frac{1}{2}x} + 3)(e^{\frac{1}{2}x} - 1) = 0$

$e^{\frac{1}{2}x} = -3$ $e^{\frac{1}{2}x} = 1$

$\times$ $x = 0$

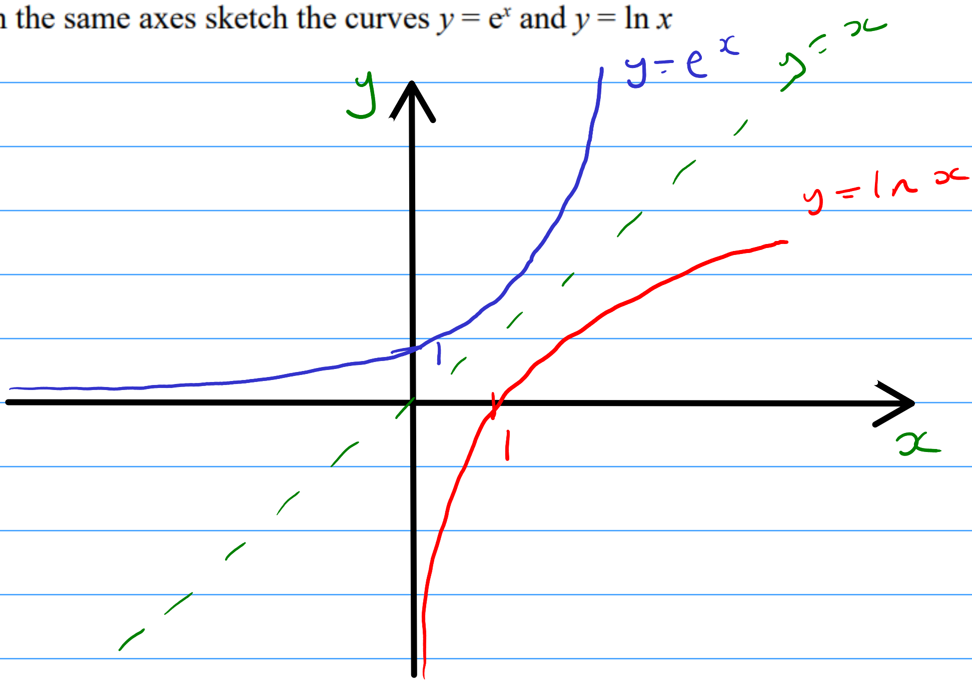
$y = 1$

$(0, 1)$

29 Find the exact solution to the equation $(2^x)^2 = 2(3^x)$

$$\ln 2^{2x} = \ln(2(3^x))$$
$$2x \ln 2 = \ln 2 + \ln 3^x$$
$$2x \ln 2 = \ln 2 + x \ln 3$$
$$2x \ln 2 - x \ln 3 = \ln 2$$
$$x(2\ln 2 - \ln 3) = \ln 2$$
$$x = \frac{\ln 2}{2\ln 2 - \ln 3}$$

30 On the same axes sketch the curves $y = e^x$ and $y = \ln x$



31 (a) Write down the value of $\log_a a$
(b) Write down the value of $\log_a a^2$

a/ 1

b/ $2\log_a a = 2$

32 (a) Show that the equation $2\log_2 x = \log_2(x + a) + 3$, can be expressed in the form $x^2 - 8x - 8a = 0$ **(3)**

(b) Given the equation $2\log_2 x = \log_2(x + a) + 3$ has only one real root find the value of a. **(3)**

a/ $$\log_2 x^2 = \log_2(x+a) + 3$$

$$\log_2 x^2 - \log_2(x+a) = 3$$

$$\log_2\left(\frac{x^2}{x+a}\right) = 3$$

$$\frac{x^2}{x+a} = 2^3$$

$$x^2 = 8(x+a)$$

$$x^2 = 8x + 8a$$

$$\underline{\underline{x^2 - 8x - 8a = 0}}$$

b/ $$b^2 - 4ac = 0$$

$$(-8)^2 - 4(1)(-8a) = 0$$

$$64 + 32a = 0$$

$$\underline{\underline{a = -2}}$$

33 Solve the equation $2\log_2(x+6) = \log_2(x+4) + 3$,

$$2\log_2(x+6) - \log_2(x+4) = 3$$

$$\log_2\left(\frac{(x+6)^2}{x+4}\right) = 3$$

$$\frac{(x+6)^2}{x+4} = 8$$

$$x^2 + 12x + 36 = 8(x+4)$$

$$x^2 + 12x + 36 = 8x + 32$$

$$x^2 + 4x + 4 = 0$$

$$(x+2)^2 = 0$$

$$\underline{\underline{x = -2}}$$

---

34 Solve the equation

$$2\log_2(x) + \log_2(\sqrt{x}) = 10$$

$$2\log_2 x + \frac{1}{2}\log_2 x = 10$$

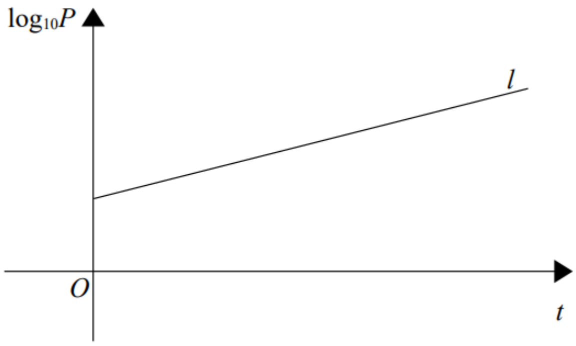
$$\frac{5}{2}\log_2 x = 10$$

$$\log_2 x = 4$$

$$x = 2^4$$

$$= \underline{\underline{16}}$$

35



The world population, P (billion), is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since 1 April 1974.

The line l illustrates the linear relationship between t and $\log_{10}P$ since 1 April 1974.
The equation of line l is $\log_{10}P = 0.008t + 0.6$

(a) Find, to 4 significant figures, the value of a and the value of b **(4)**

(b) With reference to the model interpret
  (i) the value of the constant a,
  (ii) the value of the constant b. **(2)**

(c) Find the world population, as predicted by the model, on 1 April 2020. **(2)**

(d) Given the world population on April 2020 was 7.8 billion use your answer in part (c) to comment on the suitability of the model. **(1)**

$$\log_{10}P = 0.008t + 0.6$$

$$P = 10^{0.008t + 0.6}$$

$$P = 10^{0.6}\left(10^{0.008}\right)^t$$

$$\underline{\underline{a = 3.981}} \qquad \underline{\underline{b = 1.019}}$$

b/ i/ The population in 1974 was 3.981 billion
ii/ The population increases by 1.9% each year.

c/ $P = 3.981 \times 1.019^{46}$
$= \underline{9.46}$ billion

d/ The model overestimates the population, it is not suitable.

36 The value of a car is modelled using the formula $V = 17100e^{-0.2t} + 2000$

Where V is the value of the car and the car's age is t years.

(a) Find the initial value of the car. **(1)**

Given the model predicts that the value of the car is decreasing at a rate of £1000 per year at the instant when $t = T$

(b) (i) Show that $3420e^{-0.2T} = 1000$

(ii) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month. **(6)**

a/ £19100

b/ $\frac{dV}{dt} = -3420e^{-0.2t}$

$-3420e^{-0.2T} = -1000$

$3420e^{-0.2T} = 1000$

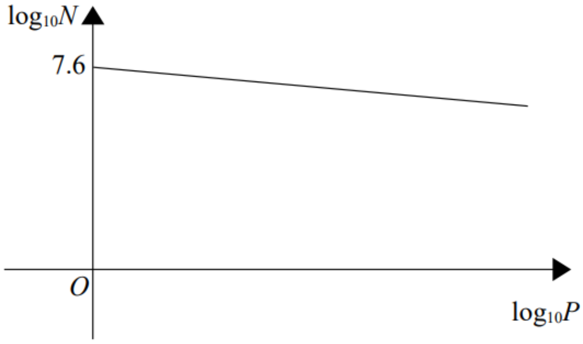
ii/ $e^{-0.2T} = \frac{50}{171}$

$-0.2T = \ln\left(\frac{50}{171}\right)$

$T = 6.15$ years

$= 6$ years 2 months

37



A company model the number of products they will sell N, and the price of the products, P, by the equation

$$N = AP^B$$

where A and B are constants.

The graph shows the linear relationship between $\log_{10}N$ and $\log_{10}P$

The line meets the vertical axis at 7.6 and has a gradient of -1.32

(a) Find, to the 3 significant figures, the value of A and the value of B. **(3)**

The company sets a selling price at £74.95.

(b) Find, according to the model, an estimate for the number of products the company will sell. **(2)**

The company estimate their production costs to be £20 per product.

(c) Estimate the profit the company will make from the product. **(2)**

a/ $\log_{10}N = -1.32\log_{10}P + 7.6$

$\log_{10}N = \log_{10}P^{-1.32} + 7.6$

$\log_{10}N - \log_{10}P^{-1.32} = 7.6$

$\log_{10}\left(\frac{N}{P^{-1.32}}\right) = 7.6$

$\frac{N}{P^{-1.32}} = 10^{7.6}$

$N = 10^{7.6}P^{-1.32}$

$\underline{A = 39800000}$ $\underline{B = -1.32}$

b/ $N = 39800000(74.95)^{-1.32}$

$= \underline{133000}$ (3sf)

c/ $133000 \times 54.95 = \underline{£7310000}$ (3sf)

38 Express as a single logarithm

$$3 \log_a 4 - \log_a 2$$

$\log_a 4^3 - \log_a 2$

$\log_a 64 - \log_a 2$

$\log_a 32$

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39 The value of an investment grew exponentially from £3.2 million in 2001 to £6.1 million in 2021.

Estimate the value of the investment in 2031 if the exponential growth continued.

$V = 3.2e^{kt}$

$6.1 = 3.2e^{20k}$

$\frac{61}{32} = e^{20k}$

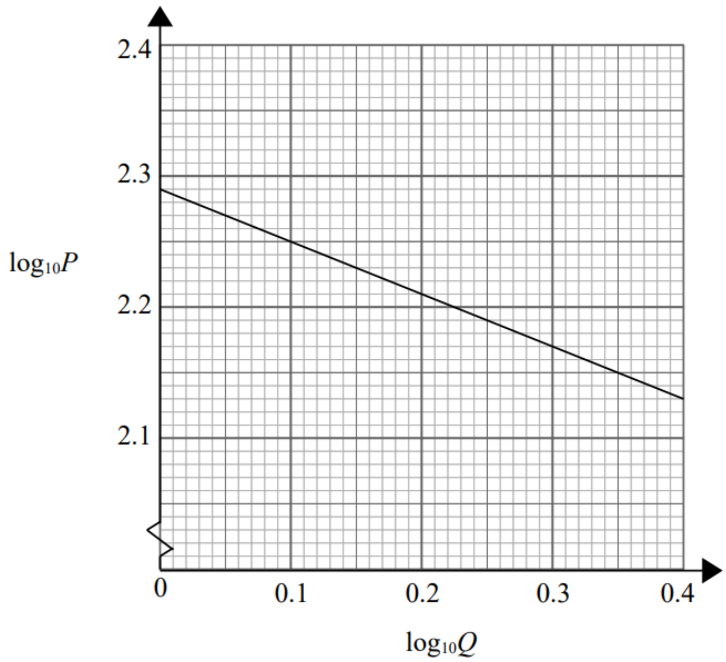
$\ln\left(\frac{61}{32}\right) = 20k$

$k = 0.0323$

$V = 3.2e^{0.0323(30)}$

$= 8.4$ million (2sf)

40



The graph shows the relationship between $\log_{10}P$ and $\log_{10}Q$.

Show that P and Q can be connected by the equation $P = aQ^b$ where a and b are constants.

$c = 2.29$ $\qquad m = \dfrac{-0.08}{0.2}$

$$= -\frac{1}{25}$$

$$\log_{10} P = -\frac{1}{25}\log_{10} Q + 2.29$$

$$\log_{10} P + \frac{1}{25}\log_{10} Q = 2.29$$

$$\log_{10} P + \log_{10} Q^{\frac{1}{25}} = 2.29$$

$$\log_{10} PQ^{\frac{1}{25}} = 2.29$$

$$PQ^{\frac{1}{25}} = 10^{2.29}$$

$$P = 10^{2.29} Q^{-\frac{1}{25}}$$

$$\underline{P = 195\, Q^{-\frac{1}{25}}}$$

41 Michael's car is valued after 2 years and after 5 years. The valuations are shown in the table below.

Time (years)	2	5
Value (£)	47200	29200

Michael models the relationship between V, the value of the car and t, the time in years since Michael bought the car, by

$$V = Ae^{-kt}$$

where A and k are constants.

(a) Explain what the value of A represents. **(1)**

(b) Show that

$$\ln V = \ln A - kt$$ **(1)**

(c) Calculate the value of A and the value of k **(4)**

(d) Use the model to predict the value of the car after 10 years. **(2)**

a/ The initial value of the car.

b/
$$v = Ae^{-kt}$$
$$\ln v = \ln(Ae^{-kt})$$
$$\ln v = \ln A + \ln e^{-kt}$$
$$\underline{\ln v = \ln A - kt}$$

c/
$$\ln 47200 = \ln A - 2k$$
$$\ln 29200 = \ln A - 5k$$

$$\ln 47200 - \ln 29200 = 3k$$
$$\ln\left(\frac{118}{73}\right) = 3k$$
$$k = \frac{1}{3}\ln\left(\frac{118}{73}\right) = \underline{0.16}$$

$$\ln A = \ln 47200 + \frac{2}{3}\ln\left(\frac{118}{73}\right)$$
$$\ln A = \ln 47200 + \ln\left(\frac{118}{73}\right)^{\frac{2}{3}}$$
$$\ln A = \ln\left(47200\cdot\left(\frac{118}{73}\right)^{\frac{2}{3}}\right)$$
$$A = 47200\cdot\left(\frac{118}{73}\right)^{\frac{2}{3}}$$
$$= \underline{£65000}$$

d/ $65000e^{-0.16(10)} = \underline{£13100}$