1 Solve, for $0 \leq x < 180°$, the equation,

$$\cos(2x + 15) = 0.3$$

Give your answers to one decimal place.

$360 - \text{Ans}$

$2x + 15 = \cos^{-1}(0.3)$

$2x + 15 = 72.54, 287.46$

$x = \underline{\underline{28.8°}}, \underline{\underline{136.2°}}$

---

2 Solve, for $0 \leq \theta < 180°$, the equation,

$$\sin(3\theta - 15) = 0.7$$

2^{nd} answer $= 180 - \text{Ans}$

Give your answers to two decimal places.

$3\theta - 15 = \sin^{-1}(0.7)$

$= 44.4°, 135.6°, 404.4°, 495.6°$

$\theta = \underline{\underline{19.81°}}, \underline{\underline{50.19°}}, \underline{\underline{139.81°}}, \underline{\underline{170.19°}}$

---

3 Solve, for $-180 \leq \theta < 180°$, the equation,

$$\tan(\theta + 30) = -2.5$$

2^{nd} answer $180 + \text{Ans}$

Give your answers to one decimal place.

$\theta + 30 = \tan^{-1}(-2.5)$

$\theta + 30 = -68.2°, 111.8°$

$\theta = \underline{\underline{-98.2°}}, \underline{\underline{88.8°}}$

---

4 Solve, for $0 \leq x < 360°$, the equation,

$$5\cos(x - 40) = 2$$

Give your answers to two decimal places.

$\cos(x - 40) = \frac{2}{5}$

$x - 40 = 66.42°, 293.58°$

$x = \underline{\underline{106.42°}}, \underline{\underline{333.58°}}$

5 Solve, for $0 \le x < 360^\circ$, the equation,

$$\tan^2(x) = 3$$

$$\tan x = \pm\sqrt{3}$$

$\tan x = \sqrt{3}$

$x = 60^\circ, 240^\circ$

$\tan x = -\sqrt{3}$

$x = -60^\circ, 120^\circ, 300^\circ$

$$x = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

---

6 (a) Show that the equation

$$2\sin^2 x = 7\cos x + 5$$

Can be written in the form

$$2\cos^2 x + 7\cos x + 3 = 0 \qquad \textbf{(3)}$$

(b) Hence solve, for $0 \le x < 360^\circ$, the equation,

$$2\sin^2 x = 7\cos x + 5 \qquad \textbf{(5)}$$

a/

$$\cos^2 x + \sin^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$2(1 - \cos^2 x) = 7\cos x + 5$$

$$2 - 2\cos^2 x = 7\cos x + 5$$

$$0 = 2\cos^2 x + 7\cos x + 3$$

b/

$$2\cos^2 x + 7\cos x + 3 = 0$$

$$(2\cos x + 1)(\cos x + 3) = 0$$

$\cos x = -\frac{1}{2}$

$x = 120^\circ, 240^\circ$

$\cos x = -3$

$\times$ no sol.

7 (a) Show that the equation

$$6\cos^2 x = 4 - \sin x$$

Can be written in the form

$$6\sin^2 x - \sin x - 2 = 0$$ **(3)**

(b) Hence solve, for $0 \leq x < 360°$, the equation,

$$6\cos^2 x = 4 - \sin x$$ **(6)**

Give your answers to one decimal place where appropriate.

a/ $6(1 - \sin^2 x) = 4 - \sin x$

$6 - 6\sin^2 x = 4 - \sin x$

$0 = 6\sin^2 x - \sin x - 2$

b/ $(3\sin x - 2)(2\sin x + 1) = 0$

$\sin x = 2/3$ $\quad\quad \sin x = -\frac{1}{2}$

$x = 41.8°, 138.2°$ $\quad\quad x = \cancel{-30°}, 210°, 330°$

$x = 41.8°, 138.2°, 210°, 330°$

---

8 Find all values for x in the interval $0 \leq x < 360°$, for which

$$2\cos^2 x - 3\sin^2 x = 14\cos x$$

Give your answers to one decimal place.

$2\cos^2 x - 3(1 - \cos^2 x) = 14\cos x$

$2\cos^2 x - 3 + 3\cos^2 x = 14\cos x$

$5\cos^2 x - 14\cos x - 3 = 0$

$(5\cos x + 1)(\cos x - 3) = 0$

$\cos x = -\frac{1}{5}$ $\quad\quad \cos x = 3$

× no sols.

$x = 101.5°, 258.5°$

9 (a) Sketch the graph of $y = \sin(x - 30)$ for x in the interval $0 \le x < 360^\circ$ **(2)**

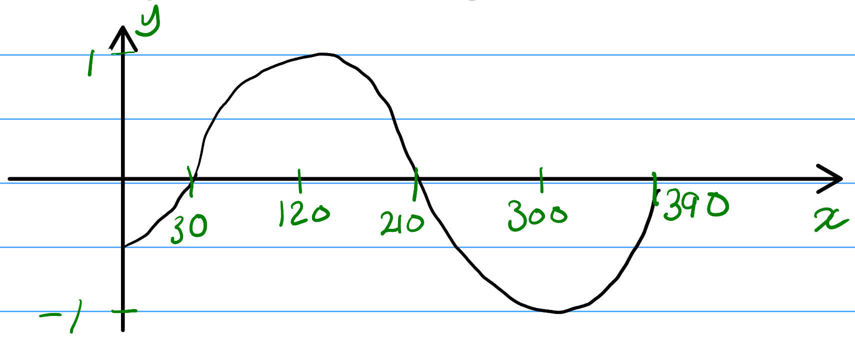
(b) Find all values for x in the interval $0 \le x < 360^\circ$, for which

$$\sin(x - 30) = 0.3$$

(4)

Give your answers to one decimal place.

a/



b/

$$\sin(x - 30) = 0.3$$

$$x - 30 = 17.5^\circ, 162.5^\circ$$

$$x = 47.5^\circ, 192.5^\circ$$

10 Find all values for x in the interval $0 \le x < 360^\circ$, for which

$$3\tan x = 4\sin x$$

Give your answers to one decimal place where appropriate.

$$\tan x = \frac{\sin x}{\cos x}$$

$$3\frac{\sin x}{\cos x} = 4\sin x$$

$$3\sin x = 4\sin x\cos x$$

$$0 = 4\sin x\cos x - 3\sin x$$

$$0 = \sin x(4\cos x - 3)$$

$$\sin x = 0 \qquad \cos x = 3/4$$

$$x = 0^\circ, 180^\circ \qquad x = 41.4^\circ, 318.6^\circ$$

11 (a) Show that the equation

$$3\sin 2x \tan 2x = \cos 2x + 2$$

Can be written in the form

$$4\cos^2 2x + 2\cos 2x - 3 = 0$$

(4)

(b) Find all values for x in the interval $0 \le x < 180°$, for which

$$3\sin 2x \tan 2x = \cos 2x + 2$$

Give your answers to two decimal places.

(6)

a/

$$3\sin 2x \cdot \frac{\sin 2x}{\cos 2x} = \cos 2x + 2$$

$$\frac{3\sin^2 2x}{\cos 2x} = \cos 2x + 2$$

$$3\sin^2 2x = \cos^2 2x + 2\cos 2x$$

$$3(1 - \cos^2 2x) = \cos^2 2x + 2\cos 2x$$

$$3 - 3\cos^2 2x = \cos^2 2x + 2\cos 2x$$

$$0 = 4\cos^2 2x + 2\cos 2x - 3$$

b/

$$\cos 2x = \frac{-1+\sqrt{13}}{4} \qquad \cos 2x = \frac{-1-\sqrt{13}}{4}$$

$$2x = 49.35°, 310.65° \qquad \text{no sols}$$

$$x = \underline{24.78°}, \underline{155.32°}$$

12 (a) Show that the equation

$$1 + \cos x = 3 \tan x \sin x$$

$\tan x = \dfrac{\sin x}{\cos x}$

Can be written in the form

$$4\cos^2 x + \cos x - 3 = 0$$

(4)

(b) Hence solve, for $0 \leq x < 360°$, the equation,

$$1 + \cos x = 3 \tan x \sin x$$

(5)

Give your answers to one decimal place where appropriate.

a/ $1 + \cos x = \dfrac{3 \sin^2 x}{\cos x}$

$\cos x + \cos^2 x = 3 \sin^2 x$ $\qquad \sin^2 x = 1 - \cos^2 x$

$\cos x + \cos^2 x = 3(1 - \cos^2 x)$

$\cos x + \cos^2 x = 3 - 3\cos^2 x$

$4\cos^2 x + \cos x - 3 = 0$

b/ $(4 \cos x - 3)(\cos x + 1)$

$\cos x = \dfrac{3}{4}$ $\qquad \cos x = -1$

$x = \underline{\underline{41.4°}}, \underline{\underline{318.6°}}$ $\qquad x = \underline{\underline{180}}$

13 (a) Show that

$$\frac{6\cos^2\theta + 7\sin\theta - 8}{1-2\sin\theta} \equiv 3\sin\theta - 2$$

(4)

(b) Hence solve, for $0 \le \theta < 360°$, the equation,

$$\frac{6\cos^2\theta + 7\sin\theta - 8}{1-2\sin\theta} = 2\cos\theta - 2$$

(3)

$$\frac{6(1-\sin^2\theta) + 7\sin\theta - 8}{1 - 2\sin\theta}$$

$\cos^2\theta = 1 - \sin^2\theta$

$$\frac{6 - 6\sin^2\theta + 7\sin\theta - 8}{1 - 2\sin\theta}$$

$$\frac{-6\sin^2\theta + 7\sin\theta - 2}{1 - 2\sin\theta}$$

$\times -1$

$\times -1$

$$\frac{6\sin^2\theta - 7\sin\theta + 2}{2\sin\theta - 1}$$

$$\frac{\cancel{(2\sin\theta - 1)}(3\sin\theta - 2)}{\cancel{(2\sin\theta - 1)}}$$

$$\underline{\underline{3\sin\theta - 2}}$$

b/

$$3\sin\theta - 2 = 2\cos\theta - 2$$

$$3\sin\theta = 2\cos\theta$$

$$3\frac{\sin\theta}{\cos\theta} = 2$$

$$3\tan\theta = 2$$

$$\tan\theta = \frac{2}{3}$$

$$\theta = \underline{\underline{33.7°}},\ \underline{\underline{213.7°}}$$

14 (a) Solve, for $360 \leq \theta < 720°$, the equation,

$$3 \cos \theta = 8 \tan \theta$$

(5)

The first four positive solutions, in order of size, of the equation

$$\cos(2a + 50) = 0.7$$

are a_1, a_2, a_3 and a_4

(b) To the nearest degree find the value of a_4.

(3)

a/
$$3 \cos \theta = \frac{8 \sin \theta}{\cos \theta}$$

$$3 \cos^2 \theta = 8 \sin \theta$$

$$3(1 - \sin^2 \theta) = 8 \sin \theta$$

$$3 - 3 \sin^2 \theta = 8 \sin \theta$$

$$0 = 3 \sin^2 \theta + 8 \sin \theta - 3$$

$$0 = (3 \sin \theta - 1)(\sin \theta + 3)$$

$$\sin \theta = \frac{1}{3} \qquad \sin \theta = -3$$

$\times$ no sols

$$\theta = 19.5°, 160.5°, 379.5°, 520.5°$$

$$\theta = \underline{\underline{379.5°}} \text{ and } \underline{\underline{520.5°}}$$

b/
$$\cos(2a + 50) = 0.7$$

1

$$2a + 50 = 45.57, 314.43, \ldots$$

$$2a_4 + 50 = 765.57$$

$$a_4 = \underline{\underline{358°}}$$

15 Solve the equation $\tan^2 2x - 3 = 0$ giving all the solutions for the interval $0 \leq x < 360°$

$$\tan^2 2x = 3$$
$$\tan 2x = \pm\sqrt{3}$$

$2x = 60°, 240°, 420°, 600°$ $\quad$ $2x = -60, 120°, 300°, 480°, 660°$

$$x = \underline{\underline{30°}}, \underline{\underline{60°}}, \underline{\underline{120°}}, \underline{\underline{150°}}, \underline{\underline{210°}}, \underline{\underline{240°}}, \underline{\underline{300°}}, \underline{\underline{330°}}$$

---

16 Given $\cos(75°) = \dfrac{\sqrt{6} - \sqrt{2}}{4}$ and $\sin(75°) = \dfrac{\sqrt{6} + \sqrt{2}}{4}$

Show that $\tan^2(75°)$ can be written in the form $a + b\sqrt{3}$

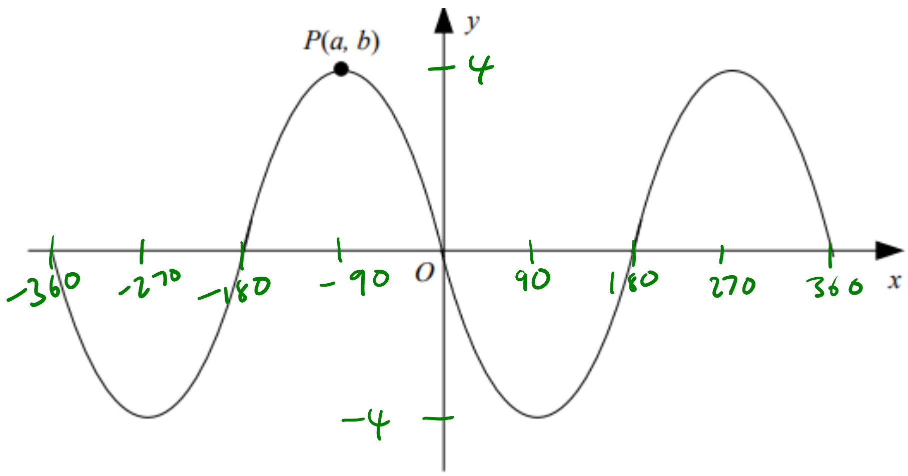
Fully justify your answer.

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\tan 75 = \frac{\sqrt{6} + \sqrt{2}}{4} \div \frac{\sqrt{6} - \sqrt{2}}{4}$$
$$= \frac{\sqrt{6} + \sqrt{2}}{\sqrt{6} - \sqrt{2}}$$
$$= 2 + \sqrt{3}$$

$$\tan^2 75 = (2 + \sqrt{3})(2 + \sqrt{3})$$
$$= 4 + 4\sqrt{3} + 3$$
$$= \underline{\underline{7 + 4\sqrt{3}}}$$

17



The graph shows part of the curve with equation $y = 4 \sin x°$

The point P is a maximum point on the curve with a being the smallest negative value of x that a maximum occurs.

(a) State the value of a and the value of b. **(1)**

(b) State the coordinates of the point to which P is mapped by the transformation which transforms the curve with equation $y = 4 \sin x°$ to the curve with equation

(i) $y = 4 \sin (x + 28)$

(ii) $y = 4 \sin (3x)$ **(2)**

(c) Solve, for $360 \leq \theta < 720°$,

$$4 \sin \theta = \tan \theta$$

Give your answers to one decimal place where appropriate. **(5)**

a/ $a = -90 \quad b = 4$

b/ i/ $(-118, 4)$

ii/ $(-30, 4)$

c/ $4 \sin \theta = \dfrac{\sin \theta}{\cos \theta}$

$4 \sin \theta \cos \theta = \sin \theta$

$4 \sin \theta \cos \theta - \sin \theta = 0$

$\sin \theta (4 \cos \theta - 1) = 0$

$\sin \theta = 0 \qquad \cos \theta = \frac{1}{4}$

$\theta = 0, 180, \underline{360}, \underline{540} \qquad \theta = 75.5°, 284.5°, \underline{435.5°}, \underline{644.5°}$

$\theta = \underline{360°}, \underline{435.5°}, \underline{540°}, \underline{644.5°}$

18 Solve $\tan 2\theta - 1 = 0$ giving all the solutions for the interval $0 \le \theta < 360°$

$\tan 2\theta = 1$

$2\theta = 45°, 225°, 405°, 585°$

$\theta = \underline{\underline{22.5°}}, \underline{\underline{112.5°}}, \underline{\underline{202.5°}}, \underline{\underline{292.5°}}$

---

19 (a) Solve $6\sin^2\theta = \cos\theta + 4$ giving all the solutions for the interval $0 \le \theta < 360°$ **(4)**

(b) Hence, hence solve $6\sin^2 2\theta = \cos 2\theta + 4$ giving all the solutions for the interval $0 \le \theta < 360°$ **(2)**

a) $\sin^2\theta = 1 - \cos^2\theta$

$6(1 - \cos^2\theta) = \cos\theta + 4$

$6 - 6\cos^2\theta = \cos\theta + 4$

$0 = 6\cos^2\theta + \cos\theta - 2$

$0 = (2\cos\theta - 1)(3\cos\theta + 2)$

$\cos\theta = \frac{1}{2}$ $\quad\quad$ $\cos\theta = -\frac{2}{3}$

$\theta = \underline{\underline{60°}}, \underline{\underline{300°}}$ $\quad\quad$ $\theta = \underline{\underline{131.8°}}, \underline{\underline{228.2°}}$

b) $2\theta = 60°, 300°, 420°, 660°$ $\quad$ $2\theta = 131.8°, 228.2°, 491.8°, 588.2°$

$\theta = \underline{\underline{30°}}, \underline{\underline{150°}}, \underline{\underline{210°}}, \underline{\underline{330°}}$ $\quad$ $\theta = \underline{\underline{65.9°}}, \underline{\underline{114.1°}}, \underline{\underline{245.9°}}, \underline{\underline{294.1°}}$

20 At 12 noon the temperature in Harry's house is 22°C
At 6 pm the temperature in Harry's house in 25°C

Harry models the temperature in his house, T, by the formula

$$T = A + B\sin(15h)$$

where h is the number of hours after 12 noon.

(a) State the value that Harry should use for A. **(1)**

(b) State the value that Harry should use for B. **(1)**

(c) Using this model, calculate the temperature in Harry's house at 9 pm. **(1)**

(d) Using the model find the number of hours in a day that the temperature will be above 23.5°C **(4)**

a/ 22

b/ $25 = 22 + B\sin(90)$

$B = 3$

c/ $T = 22 + 3\sin(135)$

$= 24.1°$ (3sf)

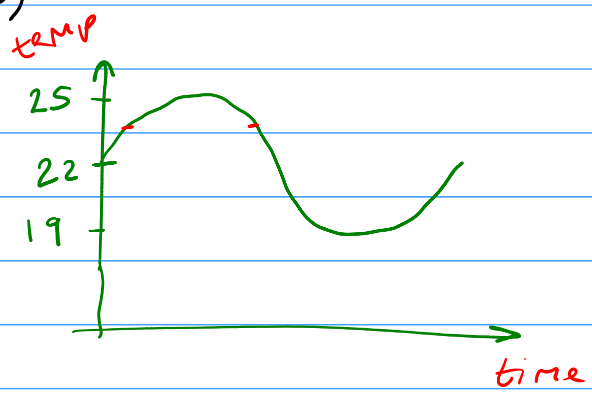
d/ $23.5 = 22 + 3\sin(15h)$

$\sin(15h) = \frac{1}{2}$

$15h = 30, 150$

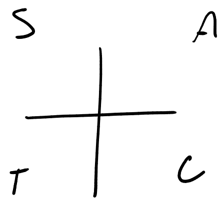
$h = 2, 10$

$10 - 2 = 8$ 8 hours above 23.5°



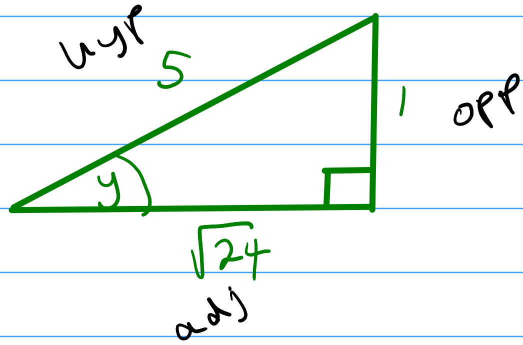
21 It is given that $\sin y = -0.2$ and $180° < y < 270°$

Find the exact value of $\cos y$



$\sin y = -\frac{1}{5}$

cos is negative between 180 and 270



$\sqrt{5^2 - 1^2} = \sqrt{24}$

$\cos y = -\frac{\sqrt{24}}{5}$

22 Jacob has to solve the equation

$$3 - \sin x = 1 + 2\cos^2 x$$

where $-180° \leq x < 180°$

Jacob's working is as follows:

$$3 - \sin x = 1 + 2\cos^2 x$$
$$2 - \sin x = 2\cos^2 x \checkmark$$
$$2 - \sin x = 2(1 - \sin^2 x) \checkmark$$
$$2 - \sin x = 2 - 2\sin^2 x \checkmark$$
$$-\sin x = -2\sin^2 x \checkmark$$
$$1 = 2\sin x \qquad 2\sin^2 x - \sin x = 0$$
$$\sin x = 0.5$$
$$x = 30°, 150°$$

(a) Explain the two errors that Jacob has made. **(2)**

(b) Write down all the values of x that satisfy the equation

$$3 - \sin x = 1 + 2\cos^2 x$$

where $-180° \leq x < 180°$ **(2)**

a/ ① Jacob should not have divided through by $\sin x$. This results in not getting the answer $\sin x = 0$

② He did not find all solutions for $\sin x = 0.5$ in the range

b/
$$2\sin^2 x - \sin x = 0$$
$$\sin x(2\sin x - 1) = 0$$

$\sin x = 0$ $\qquad$ $\sin x = \frac{1}{2}$

$x = 0°, -180°$ $\qquad$ $x = 30°, 150°$

23 Find all solutions of

$$6\cos^2 x + 5\sin x - 7 = 0$$

where $0^\circ \leq x < 360^\circ$

Give your solutions to the nearest degree.

$$6(1 - \sin^2 x) + 5\sin x - 7 = 0$$
$$6 - 6\sin^2 x + 5\sin x - 7 = 0$$
$$-6\sin^2 x + 5\sin x - 1 = 0$$
$$6\sin^2 x - 5\sin x + 1 = 0$$
$$(6\sin x + 1)(\sin x - 1) = 0$$

$\sin x = -\frac{1}{6}$ $\sin x = 1$

$x = \cancel{-9^\circ}, \underline{\underline{190^\circ}}, \underline{\underline{350^\circ}}$ $x = \underline{\underline{90^\circ}}$

---

24 (a) Show that the equation

$$2\sin^2 x = 4\cos^2 x - \cos x$$

can be expressed in the form

$$6\cos^2 x - \cos x - 2 = 0$$ **(3)**

(b) Hence, solve the equation

$$2\sin^2 2\theta = 4\cos^2 2\theta - \cos 2\theta$$

giving all values of θ between 0° and 180°, correct to 1 decimal place. **(5)**

$$2\sin^2 x = 4\cos^2 x - \cos x$$
$$2(1 - \cos^2 x) = 4\cos^2 x - \cos x$$
$$2 - 2\cos^2 x = 4\cos^2 x - \cos x$$
$$2 = 6\cos^2 x - \cos x$$
$$0 = \underline{\underline{6\cos^2 x - \cos x - 2}}$$

b/ $$6\cos^2 2\theta - \cos 2\theta - 2 = 0$$

$\cos 2\theta = \frac{2}{3}$ $\cos 2\theta = -\frac{1}{2}$

$2\theta = 48.2^\circ, 311.8^\circ$ $2\theta = 120^\circ, 240^\circ$

$$\theta = \underline{\underline{24.1^\circ}}, \underline{\underline{60^\circ}}, \underline{\underline{120^\circ}}, \underline{\underline{155.9^\circ}}$$

25 (a) Solve the equation $\sin^2 x = 0.25$ for $0° \le x < 360°$ **(3)**

(b) Solve the equation $\tan 3x = 1$ for $0° \le x < 180°$ **(3)**

a) $\sin x = \pm\sqrt{0.25}$

$= \pm \frac{1}{2}$

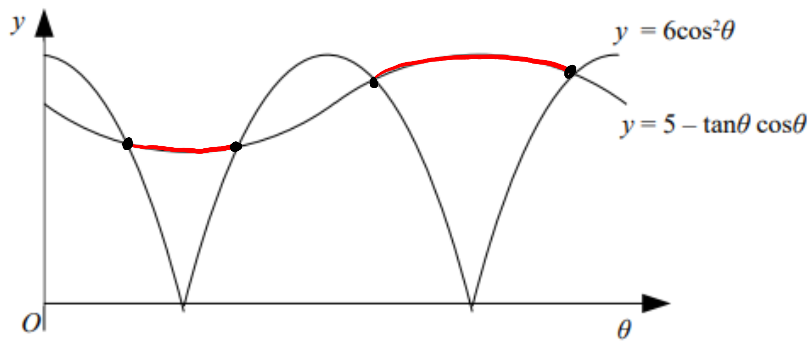
$x = 30°, 150°$ $\qquad x = -30, 210°, 330°$

b) $\tan 3x = 1$

$3x = 45°, 225°, 405°$

$x = 15°, 75°, 135°$

26 (a) Show that the equation $5 - \tan\theta\cos\theta = 6\cos^2\theta$ can be expressed in the form $6\sin^2 x - \sin x - 1 = 0$ **(2)**



The diagram shows parts of the curves $y = 6\cos^2\theta$ and $y = 5 - \tan\theta\cos\theta$, where is θ in degrees.

(b) Solve the inequality $5 - \tan\theta\cos\theta > 6\cos^2\theta$ for $0° \le \theta < 360°$ **(5)**

a) $5 - \frac{\sin\theta}{\cancel{\cos\theta}}\cancel{\cos\theta} = 6(1 - \sin^2\theta)$

$5 - \sin\theta = 6 - 6\sin^2\theta$

$6\sin^2\theta - \sin\theta - 1 = 0$

b) $\sin\theta = \frac{1}{2}$ $\qquad \sin\theta = -\frac{1}{3}$

$\theta = 30°, 150°$ $\qquad \theta = -19.5, 199.5°, 340.5°$

$30 < \theta < 150$ or $199.5 < \theta < 340.5°$

27 (a) Solve the equation $\sin^2 x = \tan^2 x$ for $0° \leq x \leq 180°$ **(5)**

(b) Prove that $\dfrac{2\sin x - \cos^2 x + 1}{2 + \sin x} \equiv \sin x$ **(3)**

a/

$$\sin^2 x = \frac{\sin^2 x}{\cos^2 x}$$

$$\sin^2 x \cos^2 x = \sin^2 x$$

$$\sin^2 x \cos^2 x - \sin^2 x = 0$$

$$\sin^2 x (\cos^2 x - 1) = 0$$

$\sin^2 x = 0$ $\cos^2 x = 1$

$\sin x = 0$ $\cos x = \pm 1$

$x = \underline{\underline{0°}}, \underline{\underline{180°}}$ $x = \underline{\underline{0°}}, \underline{\underline{180°}}$

b/

$$\frac{2\sin x - \cos^2 x + 1}{2 + \sin x}$$

$$\frac{2\sin x - (1 - \sin^2 x) + 1}{2 + \sin x}$$

$$\frac{2\sin x - 1 + \sin^2 x + 1}{2 + \sin x}$$

$$\frac{\sin^2 x + 2\sin x}{2 + \sin x}$$

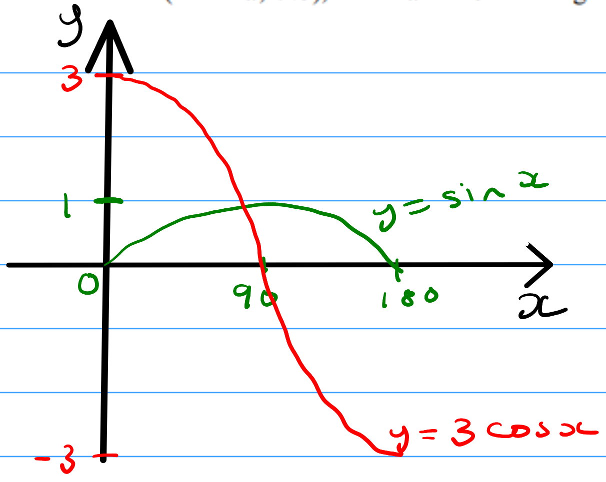
$$\frac{\sin x \cancel{(\sin x + 2)}}{\cancel{(\sin x + 2)}}$$

$$\underline{\underline{\sin x}}$$

28 (a) Sketch the graphs of $y = 3\cos x$ and $y = \sin x$ for $0^\circ \le x \le 180^\circ$ on the same axes. **(2)**

(b) Find the exact coordinates of the point of intersection of these graphs, giving the answer in the form ($\arctan a$, $k\sqrt{b}$), where a and b are integers and k is rational. **(4)**

a)



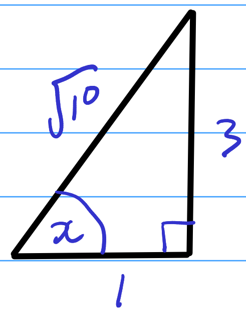
b)

$$\sin x = 3 \cos x$$

$$\tan x = 3$$

$$x = \arctan 3$$

$$\tan x = \frac{O}{A} = \frac{3}{1}$$



$$\sqrt{3^2 + 1^2} = \sqrt{10}$$

$$\sin x = \frac{3}{\sqrt{10}}$$

$$= \frac{3}{10}\sqrt{10}$$

$\therefore$ intersection at $\underline{\underline{\left(\arctan 3, \frac{3}{10}\sqrt{10}\right)}}$

29 Solve the equation $5\sin x = 3\cos x$ for $0° \le x \le 360°$

$$5\tan x = 3$$
$$\tan x = \frac{3}{5}$$
$$x = \underline{\underline{31.0°}},\ \underline{\underline{211°}} \qquad (3sf)$$

---

30 Solve the equation $24\tan x + 5\cos x = 0$ for $0° \le x \le 360°$, giving your answers to the nearest degree

$$\frac{24\sin x}{\cos x} + 5\cos x = 0$$
$$24\sin x + 5\cos^2 x = 0$$
$$24\sin x + 5(1 - \sin^2 x) = 0$$
$$24\sin x + 5 - 5\sin^2 x = 0$$
$$5\sin^2 x - 24\sin x - 5 = 0$$

$\sin x = 5$ $\qquad$ $\sin x = -\frac{1}{5}$

<u>no sols</u>

$$x = \cancel{-11.5°},\ \underline{\underline{192°}},\ \underline{\underline{348°}}$$